

# Two Essays in Finance

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# Introduction

In Chapter 1, I show that two nominal frictions, nominal debt and price stickiness, combine to create a channel for inflation to affect the real investment and leverage decisions of firms in the cross-section. With nominal debt, a positive inflation shock lowers the real value of current debt, increasing the marginal cost of adjusting debt, and thereby reduces the optimal amount of future debt. Inflation thus reduces the degree of debt overhang, increasing investment. Price stickiness leads to differences in real profitability following inflation shocks that drive cross-sectional differences in investment and leverage. In the US data, firms increase investment and reduce debt in response to positive inflation shocks, while in the cross section sticky-price firms have lower profitability and investment and higher debt than flexible firms after positive inflation shocks.

In Chapter 2, coauthored with John Boyd and Abu Jalal, we look at the effects of inflation on nominal contracting in a costly state verification framework. The nominal repayment on a private debt contract does not obey the Fisher equation and is lower than the Fisher nominal rate. We also find that higher levels of expected inflation reduce welfare by increasing the expected costs of monitoring. We document the first result empirically by looking at a panel of interest rates across countries, finding that inflation enters non-linearly. This is not true of government debt, where the Fisher equation appears to hold.



# Chapter 1

## Nominal Frictions, Firm Leverage, and Investment

### 1.1 Introduction

I show that nominal frictions, in particular nominal debt contracts and nominal goods price stickiness, help to explain variation in real firm investment and debt issuance over time and in the cross section.<sup>1</sup> The two nominal frictions that I consider here are relevant to such an analysis of leverage and investment choices. Almost all US corporate debt is denominated in dollars, which means that fluctuations in the inflation rate change the real value of debt. Price stickiness is well documented empirically and used in many models to explain macroeconomic dynamics.<sup>2</sup> Price stickiness interacts with inflation to affect a firm's leverage and investment choices by altering real profitability. Real profitability fluctuates because the firm is unable to change its prices to adjust to the new inflation environment, leaving it stuck at a nonoptimal price.

My contribution is to study the effects of these two nominal frictions jointly to

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<sup>2</sup>See Nakamura and Steinsson (2008), Kehoe and Midrigan (2015), and Weber (2016).

understand the cross-sectional implications for investment and debt issuance. This is an important topic in light of policy discussions, including from prominent economists like Kenneth Rogoff, regarding whether the Fed should be targeting higher inflation. My research sheds light on the effects that higher inflation would have on investment and leverage at the firm level, as well as the differences across industries driven by differences in price stickiness.

The mechanism that connects the nominal debt friction to leverage and investment decisions is closely related to the concept of debt overhang first discussed in Myers (1977) and more recently in Gomes, Jermann, and Schmid (2016). Gomes, Jermann, and Schmid (2016) models the interaction between inflation and long-term nominal debt on aggregate leverage, investment, output, and consumption. Debt overhang is the idea that debt can lead to suboptimal investment because some of the return from investing accrues to debt holders. When debt is in nominal terms, shocks to inflation affect the real value of debt and move the firm away from its prior leverage ratio. A positive inflation shock both lowers current real debt and leads the firm to lower its leverage the following period. The inflation shock, by reducing real debt, attenuates the debt overhang problem, leading to more real investment.

The effect of inflation on the optimal leverage decision comes from the fact that an inflation shock reduces the real value of current nominal debt. Thus, when the firm considers issuing to reach a future debt level, more debt must be issued to reach that level than in the case where there is no inflation shock. There is an endogenous cost associated with issuing additional debt: issuing another dollar of debt reduces the price of the debt because the chance of default rises. This reduction in price affects all new debt issued. The level the firm would have chosen absent the inflation shock is now too high, since to reach it the firm would have to issue more debt and face a larger cost from the price reduction, and the optimal amount of future leverage decreases.

Variation in real profitability also affects optimal leverage and investment. If firms

cannot change prices despite changes in inflation, and if the firm’s optimal selling price depends on the overall inflation rate, as is the case in sticky price models, then real profitability will vary with inflation in a systematic way. These differences in real profitability then cause firms to make different investment and leverage choices. If real profitability falls with inflation, for example, then lower cash flows can lead to a higher chance of default and less debt relative to the case where profitability is unaffected by inflation. Less debt means more investment because debt overhang is reduced. However, lower real profitability can also lead the firm to reduce investment, so the result for investment is less clear.

I test the model’s assumptions and predictions empirically. The data generally bears out the predictions of the model. Flexible firms have higher profitability, as measured by EBIT margin or return on assets, after positive inflation shocks. Investment increases with inflation, and firms with more flexible prices invest more when inflation is high. On the leverage side, positive inflation shocks reduce firm leverage at the quarterly time frame. In contrast to the model, which suggests small differences in leverage across firms with different inflation sensitivity, the data suggests that flexible firms reduce leverage more in response to inflation.

I also consider whether inflation and deflation shocks matter differently for firms, as it is unclear a priori if price stickiness leads to a symmetric relationship between inflation and real profitability. To do so, I run the same regressions of profitability, investment, and leverage on inflation shocks interacted with price stickiness, but I separate the inflation shocks and interaction terms based on the sign of the inflation shock in that period. The results suggest that the connection between inflation and these variables is stronger for deflation shocks.

I use a version of Weber (2016)’s sticky price measure. He provided me with price stickiness data aggregated at the six-digit NAICS industry level. I use the personal consumption expenditure (PCE) inflation data for the same time period as my measure of inflation. Because firms may anticipate future inflation, my main

specification uses unexpected inflation shocks in addition to actual inflation. Results are similar using CPI as the inflation measure.

Panel regressions show that inflation shocks affect investment and leverage, as well as profitability. High inflation reduces debt and increases investment, in line with the theory. I also find that the interaction terms between price stickiness and inflation are significant, suggesting that firms respond differently to inflation depending on their price stickiness. In terms of magnitude, for a firm with complete price flexibility, a 1% decrease in quarterly inflation translates to a 0.5-1.5% decrease in investment and 1.0-2.5% increase in leverage. For a firm with completely sticky prices, the effects are 0.5-1.0% smaller for investment and 1.0-2.0% for leverage respectively, so that sticky-price firms are less sensitive to inflation.

These effects are stronger for deflationary periods. When I consider inflation and deflation shocks separately, a 1% deflation shock raises firm leverage 1.5-3.0% for flexible firms, and 0-0.5% for sticky-price firms, whereas a 1% positive inflation shock lowers leverage 0-2.5%, and the coefficients are often not statistically significant. For investment, a 1% deflation shock lowers investment 0.5-1.5% for flexible firms, while sticky-price firms hardly react. Positive inflation shocks do not affect investment as much.

I also document that there are significant differences in how firms react to inflation shocks depending on the firm's leverage and debt maturity, as these drive the effect inflation has on optimal leverage, and thus investment. Firms with greater debt maturity see larger changes in leverage and investment following inflation shocks. Firms with higher leverage see greater increases in investment resulting from the larger reduction in debt overhang.

In the rest of the paper, Section 1.2 further discusses the related literature and this paper's contribution. Section 1.3 details the model and theoretical results. Section 1.4 explains the data and methodology. Section 1.5 shows the main empirical results regarding leverage and investment. Section 1.6 contains additional empirical results

on the effects of debt maturity on firm decisions, the effect of leverage on investment, and robustness tests. Section 1.7 concludes.

## 1.2 Related literature

This paper contributes to the large literature in finance explaining firm investment and debt issuance decisions, including Modigliani and Miller (1958), Myers (1974), Lang et al. (1996), Hennessy (2004), Tsyplakov (2008), and Sundaresan et al. (2015), among others. My paper differs from other papers that explain real leverage and investment decisions by introducing nominal frictions as a factor driving these choices. Other papers have explored the effects of real macroeconomic fluctuations on investment choices, including Chen and Manso (2014); I focus on nominal effects.

My work is most closely related to a recent paper, Gomes, Jermann, and Schmid (2016).<sup>3</sup> That paper explores the effects of nominal debt on aggregate investment and leverage. In their model, all firms make the same decisions and there is no heterogeneity in nominal frictions. They show that even i.i.d. inflation matters for real decisions regarding leverage and investment. I extend their model to include heterogeneity across firms, and furthermore test the predictions of the model, and the extension, in the data. In this paper, firms are organized into industries that differ on their degree of nominal price stickiness. Exposure to inflation therefore comes from two sources, nominal debt and nominal price stickiness. This leads to differences in how firms react to changes in inflation based on the degree of price stickiness they exhibit and their amount of leverage.

There is a substantial literature connecting inflation to leverage. Modigliani (1982) links positive inflation to higher values of leverage. Taggart (1985) shows that higher expected inflation can increase leverage under certain conditions, as firms benefit more from the tax deductibility of interest. Frank and Goyal (2009) document that leverage

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<sup>3</sup>Miao and Wang (2010) and Occhino and Pescatori (2012) also explore the implications of corporate debt in a business cycle model.

is positively associated with expected inflation. My paper differs in that it considers short-term actual and unexpected inflation shocks, not expected inflation, and shows that cross-sectional exposure to inflation via price stickiness also matters for leverage decisions. The channel in my paper is via nominal debt contracts themselves, rather than nominal interest payments. Kang and Pflueger (2015) study the effects of debt deflation on corporate bond yields. Eraker et al. (2016) model the effects of inflation on stocks and bonds in an economy with durable and nondurable sectors.

Models of sticky prices have a long history in the macroeconomic literature, starting with Taylor (1980) and Calvo (1983). There is significant evidence that firms face frictions in terms of changing prices and wages (Nakamura and Steinsson (2008), Kehoe and Midrigan (2015), and Weber (2016)). There is also a large macroeconomic literature that uses nominal rigidities, nominal debt, and inflation to explain aggregate movements in consumption, leverage, and investment starting from Fisher (1933), and including more recently Sveen and Weinke (2005), Sveen and Weinke (2007), Altig et al. (2011), and Eggertsson and Krugman (2012), among others. While my paper draws heavily from these macroeconomic ideas, it is focused on the firm-level consequences of nominal rigidities.

I also contribute to a recent literature exploring the implications of nominal and real rigidities in a finance context. Li and Palomino (2014) model production with nominal price and wage rigidities and show those frictions lead to higher equity returns. Weber (2016) finds that companies with higher price stickiness earn higher returns, while Gorodnichenko and Weber (2016) show that the stock prices of sticky-price firms are more volatile following monetary policy shocks. Duarte (2013) performs rolling regressions of real stock returns on inflation, finding that stocks with lower inflation betas earn higher returns. Smith and van Eegteren (2005) relate inflation to investment via a retained earnings channel, where inflation reduces the purchasing power of retained earnings and as a consequence lowers investment. In contrast to this paper, I find that inflation increases investment. While I don't model retained earnings directly, the model does extend to cases where net debt is positive. Other

work in the area includes Favilukis and Lin (2014), which explains several asset pricing puzzles by introducing real wage stickiness into a production model. D’Acunto et al. (2015) show that sticky price firms have lower leverage and suggests that sticky price firms are riskier and more likely to default. That paper also shows that improved monitoring allows sticky-price firms to reduce their risk and issue more debt. De Fiore et al. (2011) shows that monetary policy producing inflation volatility improves outcomes in a model where firms have nominal retained earnings and debt.

My work contributes to the literature by linking inflation, leverage, and investment together. Furthermore, I show that sticky prices also matter for profitability and investment rates. Importantly, I show that these connections are conditional on current inflation shocks. That is, firms react differently to inflation shocks depending on their price stickiness. This conditional result is new to the literature. In this paper, there are no retained earnings, as all profits are paid out as dividends, which can be negative if the firm needs to issue equity. Firms are also exposed to inflation via profitability. My model also only considers real investment, with debt the only nominal quantity.

## **1.3 Model**

In this section, I present a model of firm investment and debt issuance where firms are influenced by nominal frictions.

### **1.3.1 A dynamic model**

I model an economy with two sectors, both of which produce the same consumption good. The sectors differ in their profit sensitivity to inflation, which is a reduced-form way of modeling differing levels of nominal price stickiness in the two sectors. Having two sectors allows me to generate cross-sectional differences in the effect of inflation on firm decisions. The model is similar to that presented in Gomes et al. (2016) with the addition of a second production sector and profit sensitivity to inflation.

## Production function and profitability

A firm in sector  $j$  chooses next period's investment and debt so as to maximize firm value. In each period, firms produce profit, invest, and adjust their debt structure.

Firms produce output  $y$  according to a standard Cobb-Douglas production function  $y = f(k, l) = \beta(\mu)Ak^\alpha l^{1-\alpha}$ , where  $A$  is an aggregate productivity shock,  $k$  is capital, and  $l$  is labor. I allow real production to depend on inflation  $\mu$  through the term  $\beta(\mu)$ , which I use as a reduced-form way of representing nominal frictions in production related to sticky prices. In sticky price models (a good reference is the appendix to Weber (2016)), firms can only update their prices occasionally. As a result, they choose their price to maximize profits in the event they are unable to ever change prices again. This optimal price will depend on the degree of price stickiness and the inflation rate in the economy, and will lead to the firm's real profitability also depending on inflation.

Firms choose labor in each period to maximize profits, leading to the standard form for profit of  $\pi = R(A, \mu)k = \alpha y$ . Importantly,  $R(A, \mu)$  depends on  $A$  and  $\mu$  and includes the parameters driving cross sectional differences in the sensitivity of firm profits to inflation. Firms are also subject to a random idiosyncratic additive profitability shock  $z$ , scaled by capital, that reduces profit, so that high realizations of  $z$  are bad for the firm. (This can also be considered a firm-specific fixed cost that scales with capital.) Including this shock, firm profits are  $(R(A, \mu) - z)k$ . This  $z$  shock is independent across firms and across time and has support  $Z = [z_L, z_H]$ , density function  $\varphi(z)$ , and CDF  $\Phi(z)$ . This shock creates heterogeneity across firms within each sector and causes some firms to default. Firms are taxed at rate  $\tau$ , so after-tax profits are  $(1 - \tau)(R(A, \mu) - z)k$ .

Firms also invest in their capital stock subject to quadratic capital adjustment costs. The total cost of investment is given by  $H(i, k) = ik + \frac{\eta}{2}(i)^2 k$ , where  $i$  is the investment rate as a fraction of capital. Capital accumulation follows  $k' = (1 - \delta)k + ik$  where  $\delta$  is the depreciation rate. Primes denote next period values, and I use a minus symbol for values from prior periods.



## Debt issuance

Firms also adjust their nominal debt in each period. At the beginning of the period, firms have quantity of nominal debt  $B$ , denominated in dollars, which has a coupon rate  $c$ . A fraction  $\lambda < 1$  of debt matures each period (so debt maturity is  $1/\lambda$ ), and firms issue new debt such that the total quantity of nominal debt in the next period is  $B'$ . Converting to real quantities by dividing by the current price level  $P$ , the real change in debt in a given period is  $(B' - (1 - \lambda)B) / P$ , which can be rewritten as

$$(B' - (1 - \lambda)B) / P = \left( \frac{B'}{P} - (1 - \lambda) \frac{B}{P^-} \frac{P^-}{P} \right) \equiv \left( b' - (1 - \lambda) \frac{b}{\mu} \right)$$

where lower case  $b = B/P^-$  represents a real quantity, and  $\mu \equiv P/P^-$  is the gross inflation rate.

## Firm value

Firms have limited liability and default when their equity value falls to zero. As a result, the firm's equity value can be written as the sum of current period profits less debt servicing costs plus the firm continuation value, bounded below by zero:

$$Eq(b, k, A, \mu, z) = \max \left\{ 0, (1 - \tau)(R(A, \mu) - z)k - ((1 - \tau)c + \lambda) \frac{b}{\mu} + V(b, k, A, \mu) \right\} \quad (1.3.1)$$

where  $V(b, k, \mu)$  represents the continuation value of the firm as a function of current debt, capital, and the inflation rate. The firm chooses future capital and debt to maximize  $V$ , as shown below.  $V$  represents the net proceeds from debt issuance or retirement (the price of debt  $p(b', k')$  times the change in debt), less investment and associated adjustment costs  $H(i, k)$ , plus the tax benefits from depreciation, and next

period's expected equity value:

$$\begin{aligned}
V(b, k, A, \mu) = \max_{i, k', b'} & \left\{ p(b', k') \left( b' - (1 - \lambda) \frac{b}{\mu} \right) - H(i, k) + \tau \delta k \right. \\
& \left. + E \left[ M' \int_Z E q(b', k', A', \mu', z') d\Phi(z') \right] \right\} \\
\text{s.t. } & k' = k(1 - \delta) + ik
\end{aligned} \tag{1.3.2}$$

$$\tag{1.3.3}$$

The firm discounts future equity value by the stochastic discount factor  $M$ , which is derived from the consumer side of the economy shown below.

Equation 1.3.1 implies a default threshold in  $z$ . The firm defaults whenever  $z > z^*(A, \mu)$ , where

$$z^*(A, \mu)k = R(A, \mu)k - \left( \frac{(1 - \tau)c + \lambda}{1 - \tau} \right) \frac{b}{\mu} + \frac{V(b, k, A, \mu)}{1 - \tau} \tag{1.3.4}$$

When a firm defaults, it loses a fraction  $\xi$  of its capital as default costs and reorganizes. In the model, firms reorganize immediately and reenter the economy in the following period. Default exists in the model to create a trade-off for the leverage decision. Without default, firms would issue as much debt as possible to take advantage of the tax deduction of interest.

### Price of debt

Investors value the firm's debt by appropriately discounting expected future cash flows. The total market value of debt  $p(b', k')b'$  is equal to the expected repayments in the no-default and default regions:

$$\begin{aligned}
p(b', k')b' = E & \left[ M' \left( \Phi(z^{*'}) (c + \lambda + (1 - \lambda)p') \frac{b'}{\mu'} z \right. \right. \\
& \left. \left. + \int_{z^{*'}}^{z^H} \left( (1 - \tau)(R' - z')k' + (1 - \lambda) \frac{p'b'}{\mu'} + V(b', k', A', \mu') - \xi k' \right) d\Phi(z') \right) \right]
\end{aligned} \tag{1.3.5}$$

The first term in the equation is the bond value if the firm does not default. The

debt holder receives the coupon and fractional principal payment, plus the future value of the non-retired debt. The second term, when the firm defaults, is expected future profits plus the firm's continuation value less default costs.

The price of debt  $p(b', k')$  is a key component of the model in that it generates an endogenous debt adjustment cost that limits the firm's optimal change in debt in response to an inflation shock. This is because changing the amount of debt issued  $b'$  affects the price of debt and therefore the total proceeds of issuance. I explain this in more detail in the following sections.

### The consumer

The focus of the model is to understand the firm's decision on investment and debt. Therefore the consumer side of the model is kept as simple as possible. There is a representative consumer with CRRA utility over consumption and time worked:

$$U = E \left[ \frac{(c^{1-\theta}(1-n)^\theta)^{1-\sigma}}{1-\sigma} + \beta U' \right]$$

Time preference is given by  $0 < \beta < 1$ , risk aversion is given by  $\sigma > 0$ , while  $\theta$  is related to the elasticity of labor supply. The consumer provides labor to both sectors, owns the firm's equity and debt, and receives the tax on corporate profits. The pricing kernel  $M$  used in the debt price and value function equations is derived from the consumer's utility maximization problem.

### Market clearing

All firms in an industry make the same leverage and investment choices ex ante.

Aggregate output is the total output in each industry, less losses due to bankruptcy:  $Y = \sum_j [y_j - (1 - \Phi(z_j^*))\xi k_j]$ .

Good and labor markets clear:  $Y = c + \sum_j i_j k_j$  and  $n = \sum_j l_j$ .

### 1.3.2 Normalizing the model

The model is set up so that its key components can all be normalized by capital, reducing the number of state variables. Define leverage  $\omega \equiv b/k$ , with normalized functions denoted with lower case. Then, again following Gomes et al. (2016) I can write,

$$eq(\omega, A, \mu, z) = \max \left\{ 0, (1 - \tau)(R(A, \mu) - z) - ((1 - \tau)c + \lambda) \frac{\omega}{\mu} + v(\omega, A, \mu) \right\}$$

with

$$\begin{aligned} v(\omega, A, \mu) = & \max_{i, \omega'} \left\{ p(\omega') \left( \omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) - h(i) + \tau \delta \right. \\ & \left. + g(i) E \left[ M' \int_Z eq(\omega', \mu', A', z') d\Phi(z') \right] \right\} \end{aligned} \quad (1.3.6)$$

The ratio of next period capital to today's capital is  $g(i) \equiv k'/k = 1 - \delta + i$ . The investment rate including adjustment costs is  $h(i) = i + \frac{\eta}{2} (i)^2$ .

The optimal default threshold for  $z$  is

$$z^*(A, \mu) = R(A, \mu) - \left( \frac{(1 - \tau)c + \lambda}{1 - \tau} \right) \frac{\omega}{\mu} + \frac{v(\omega, A, \mu)}{1 - \tau}$$

and, dividing through by  $k'$ , the price of scaled debt is now

$$\begin{aligned} p(\omega') \omega' = & E \left[ M' \left( \Phi(z^{*'}) \frac{c + \lambda + (1 - \lambda)p'}{\mu'} \omega' \right. \right. \\ & \left. \left. + \int_{z^{*'}}^{z_H} \left( (1 - \tau)(R' - z') + (1 - \lambda) \frac{p' \omega'}{\mu'} + v(\omega', \mu') - \xi \right) d\Phi(z') \right) \right] \end{aligned} \quad (1.3.7)$$

### 1.3.3 First order conditions

The first order conditions (FOCs) that come from optimizing equation (1.3.6) are instructive. Detailed derivations can be found in the Model derivations appendix.

The equation for optimal investment is

$$1 + \eta i = p(\omega')\omega' + \mathbb{E} \left[ M' \int_Z eq(\omega', \mu', A', z') d\Phi(z') \right] \quad (1.3.8)$$

This equates the marginal cost of investment on the left, including adjustment costs, with the marginal benefits of additional investment on the right: the ability to add to leverage and get the extra value of issuance, and the additional profit from expanding capital.

The FOC for leverage is

$$0 = \frac{\partial p}{\partial \omega'} \left( \omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) + p(\omega') g(i) + \frac{\partial}{\partial \omega'} g(i) \mathbb{E} \left[ M' \int_Z eq(\omega', \mu', A', z') d\Phi(z') \right]$$

The last term can be simplified (see the Model derivations appendix for details):

$$p(\omega') g(i) = - \frac{\partial p}{\partial \omega'} \left( \omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) - (1 - \tau) g(i) \mathbb{E} \left[ M' \Phi(z^{*'}) \frac{\partial z^{*'}}{\partial \omega'} \right] \quad (1.3.9)$$

This equates the marginal benefit of additional leverage, the issue price on the left, with issuance's associated marginal costs on the right. The first term on the right hand side is the amount lost on the entire issuance associated with the change in price coming from increasing investment. This is in effect an endogenous adjustment cost for leverage: it is convex and depends on the size of the change in leverage, which in turn depends on the inflation shock. The second term is the additional cost of issuance associated with repaying the extra leverage and the increase in the chance of default.

### 1.3.4 Calibration

I calibrate the model with two sectors representing firms with low and high price stickiness. I target aggregate moments including average leverage, investment, and default rates, as well as sector specific leverage and investment rates. I start from the calibration in Gomes et al. (2016). That paper targets default rates and average

leverage ratios. Parameter values are given in Table 1.1. While the key model results go through even for i.i.d. inflation shocks, I assume inflation  $\mu$  and aggregate productivity  $A$  follow a joint AR(1) process to match the data.

This AR(1) process is calibrated to quarterly productivity and inflation; I use the values found in Gomes et al. (2016). I model the density of the idiosyncratic profitability shock  $z$  as a quadratic function of the form  $\varphi(z) = n_1 + n_2 z^2$ . The parameter  $n_2$  is pinned down by the choice of  $n_1$ .

For price stickiness, I parametrize  $\beta(\mu) = 1 + m_P \max\{\mu - \bar{\mu}, 0\} + m_N \min\{\mu - \bar{\mu}, 0\}$ , where  $\bar{\mu}$  is the steady-state level of inflation. This functional form captures the intuition that changes in inflation away from the steady state can affect real profitability. I allow for the response to differ depending on whether there is a positive or negative inflation shock. I perform some exploratory empirical analysis to see how inflation affects profitability. As documented in Section 1.5, I find that inflation increases real profitability more for flexible firms than sticky-price firms, and that there is an asymmetric response to inflation versus deflation shocks. Based on these empirical findings, I consider two scenarios for  $\beta(\mu)$ , one where the reaction to inflation is symmetric for flexible-price firms ( $m_P = m_N$ ), and one where only negative inflation shocks decrease profitability for flexible-price firms ( $m_P = 0, m_N > 0$ ).

### 1.3.5 Model results

There are two key results from the model: inflation shocks affect real leverage and investment, and there are significant cross-sectional effects relating to firms' profit sensitivity to inflation. I discuss them in detail below.

**Prediction 1.** A positive unexpected inflation shock leads to a reduction in firm leverage.

In the simplest case, consider i.i.d. inflation. With this assumption,  $\mu$  has no effect on the first order condition for leverage in equation 1.3.9 except through the first term on the right hand side. The optimal level of leverage equates the marginal benefits of debt issuance with the marginal costs. The marginal benefit of issuing

slightly more debt is that the firm receives the incremental debt price  $p$ . There are two costs of issuance: the additional repayment and chance of default next period, and the effect of issuance today.

When the firm chooses to issue slightly more, that lowers the price that the firm receives for the debt because  $\frac{\partial p}{\partial \omega'} < 0$ . The firm receives that lower price on *all* the debt it issues, rather than the marginal dollar.

With an unexpected positive inflation shock, the current real leverage for the firm declines. This implies that the marginal cost associated with issuing increases, as the firm loses the change  $\frac{\partial p}{\partial \omega'}$  on a greater amount of debt issuance. With i.i.d. inflation, there is no change in the marginal benefit of issuance as the next period is unaffected by the inflation shock today. As a result, optimal leverage decreases to re-equalize the marginal benefit and cost.

This mechanism implies that leverage does not fully return to its previous level following an inflation shock. Because leverage is costly to adjust via the endogenous adjustment cost described above, the firm does not end up back at its prior leverage ratio.

**Prediction 2.** A positive unexpected inflation shock leads to an increase in firm investment.

As a result of the reduction in optimal leverage, firms invest more. Lower leverage means a lower chance of default, which reduces the size of the debt overhang problem and allows the firm to invest more.

**Prediction 3.** Changes in leverage and investment depend on the firm's profit sensitivity to inflation.

If profitability falls as a result of the positive inflation shock (for example, if the firm cannot reoptimize due to price stickiness and is no longer at its profit-maximizing price), then the increase in investment stemming from the reduction in leverage will not be as large. Changes in real profitability due to inflation will also affect the firm's default threshold, and consequently, its optimal leverage. Lower real profitability following an inflation shock reduces the firm's ability to repay debt.

**Prediction 4.** Firms with longer debt maturity are more sensitive to inflation.

In the model, debt maturity, parametrized by  $\lambda$ , is exogenous. In the case of short-term debt, where  $\lambda = 1$ , and all debt matures each period, inflation shocks do not affect the leverage decision. With short-term debt, no debt rolls over. The model suggests that firms with longer maturity should be more sensitive to inflation all else equal. This is because a longer debt maturity exposes a higher fraction of the debt to inflation fluctuations and makes it harder for the firm to adjust leverage, as the marginal cost of issuing debt increases. Therefore, changes in leverage and investment following an inflation shock should be greater for firms with higher debt maturities.

### 1.3.6 Results from the calibration

The model produces interesting implications for the effect of inflation on real firm decisions. When the sectors are identical, and have no variation in profit sensitivity to inflation, the model reduces to Gomes et al. (2016) and the results are similar to those in that paper. When the two sectors vary in inflation sensitivity, shocks to the rate of inflation create differential responses in the sectors.

Motivated by empirical results regarding the effects of inflation on real profitability which I discuss below in Section 1.5, I consider two cases. First, I set  $\beta(\mu)$  to be a linear function of  $\mu$  so that a positive inflation shock increases profitability for the flexible sector relative to the sticky price sector. Second, I allow profitability to react asymmetrically to the inflation shock, and consider the effects of inflation versus deflation shocks.

Figure 1.1 shows impulse response functions for book and market leverage and investment for both sectors when the model is hit with an inflation shock. When the economy is hit by a positive inflation shock, both sectors see their book and market leverage fall through the nominal debt channel. The two sectors have different reactions to the inflation shock, because the shock affects future profitability, and thus the probability of default, differently in each sector.

Following a positive inflation shock, the sector with sticky prices does not see



profitability rise as much as it cannot adjust to the new inflation environment. Profitability in the sector with flexible prices is higher, consistent with the empirical data. As a result of the reduction in debt overhang, both sectors invest more. However, because of the hit to profitability, the sticky-price sector invests less than the flexible sector.

Figure 1.2 shows similar impulse response functions for profitability, book and market leverage, and investment when there is an asymmetric response in profitability to inflation.

When the economy is hit by a positive inflation shock, there are no effects on real profitability. As a result, the only mechanism in effect is that of Gomes et al. (2016). The inflation shock reduces the firm’s current leverage and leads to lower future leverage and higher investment. There is no difference across the sectors because both have the same profitability.

When the economy is hit by a negative inflation shock, the two sectors respond differently. Both see leverage increase as a result of the deflation shock, following the model results. As a consequence, investment falls for both sectors, but falls further. The profitability differential also leads to differences in how book and market leverage respond.

## 1.4 Data and methodology

I take the theoretical implications of the model and test them in the data. I look for links between price stickiness and inflation shocks, and leverage, investment, and profitability.

### 1.4.1 Firm-level accounting data

Firm income statement and balance sheet data is quarterly from Compustat. The sample covers 1975-2015. I perform the standard data cleaning procedures. See the data appendix for details on variable creation. My main dependent variables are

measures of profitability, investment rates, and leverage.

My measures of profitability are return on assets, operating income divided by total assets, and EBIT margin, earnings before interest and taxes over total sales.

I use two measures of investment. The first measure is capital expenditures less sales of property, plant and equipment in a given period, while the second measure considers changes in real property, plant, and equipment, including depreciation. In each quarter, I smooth the investment rate by taking the average real investment over the following four quarters divided by the prior quarter's total assets. Smoothing helps remove some of the lumpiness of investment, especially at the quarterly frequency I use.

For leverage, I consider both book and market measures. These measures are ratios of nominal variables, so provided that the same price deflator is appropriate for both the numerator and denominator, these ratios are real. Book leverage is total debt (debt in current liabilities plus long-term debt) divided by total book assets. Market leverage is the total market value of debt divided by the sum of debt and equity market values. For debt prices, I use bond transaction and pricing data from Lehman, FISD, and TRACE to calculate average quarter-end prices for debt for each firm, and use this debt price to adjust the book value of debt.

My main independent variables are the quarterly inflation shocks and a measure of firm inflation sensitivity, described in the next two subsections. I include the standard controls found in the leverage and investment literature. These include firm size, book-to-market ratio, return on assets, lagged investment, average interest rate, capital intensity, and debt maturity.

### **1.4.2 Inflation data**

I use several measures of quarterly inflation rates for robustness. I use both the BEA's personal consumption expenditures price index (PCE) and the BLS's consumer price index (CPI). The PCE measure differs from the CPI by estimating weights for goods using business rather than consumer surveys, and also adjusts for short-term changes

in the consumption basket. The PCE is my preferred measure.

While the model results depend only on the actual inflation rate, and all agents in the model know the inflation process, I also consider the effects of unexpected inflation shocks, as agents in the economy might anticipate future inflation and adjust accordingly. I use two methods to calculate unexpected inflation shocks. First, I use data from the Survey of Professional Forecasters (SPF), available from the Philadelphia Fed.<sup>4</sup> SPF provides quarterly CPI forecasts starting in 1981, and PCE starting in 2007. I construct unexpected inflation shocks by subtracting the one quarter ahead median estimate of inflation from that quarter's actual inflation rate. I also estimate unexpected inflation using the residuals of a vector autoregression of inflation on lagged inflation, GDP growth, and interest rates. Figure 1.3 shows a plot of actual and unexpected inflation rates for both PCE and CPI measures. Actual inflation is highest during the 1970s, while unexpected inflation tends to be more volatile during recessions and economic crises. Because the SPF PCE forecast only begins in 2007, I focus the results on the VAR unexpected PCE inflation shocks. Results are similar for actual and SPF unexpected inflation shocks and for CPI variants.

### 1.4.3 Industry-level price stickiness data

For my empirical analysis, I use industry-level price stickiness data. The data is from Weber (2016), and a detailed description of the data is available there. Briefly, Weber (2016) uses confidential data on firm-level price changes from the BLS producer price index (PPI) survey. For each firm he constructs a constant measure of price change frequency by dividing the number of price changes the firm registers by the number of months that firm is in the sample. He helpfully supplied me with data aggregated at the NAICS six-digit level. I subtract this frequency of price change from 1 to construct a measure of price stickiness so that firms with a high price update frequency have low price stickiness. In my sample, firms vary on this price stickiness

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<sup>4</sup><https://www.philadelphiafed.org/research-and-data/real-time-center/survey-of-professional-forecasters/data-files>

measure from 0.12 to 0.96. Table 1.2 provides summary statistics for the sticky price measure across the Fama-French 12 industries.<sup>5</sup> In general, services have higher price stickiness, while commodities and retail have lower price stickiness. This measure of price stickiness is highly correlated with industry-level PPI volatility (-0.87), which increases my confidence in its validity.

## 1.5 Empirical results

In this section, I document my main empirical results regarding the effects of inflation and price stickiness on profitability, investment, and leverage.

### 1.5.1 Inflation sensitivity and profitability

I first check to see if firm price stickiness is associated with reduced profitability, as I assume in the model. This tests one of the main assumptions in the paper, that real profitability varies with inflation as a result of nominal rigidities. I look at two measures of profitability: return on assets and EBIT margin. The intuition is that inflation shocks should hurt the profitability of sticky-price firms relative to flexible firms, as they cannot adjust to the new conditions as quickly as flexible firms.

Consistent with the intuition, I find in Table 1.3 that inflation shocks and price stickiness do in fact matter for profitability. The interaction term between inflation and price stickiness is negative, so that a positive inflation shock leads to a decrease in ROA by about 2% for sticky-price firms relative to flexible firms (the same inflation shock results in a decrease in EBIT of about 20% between sticky-price and flexible firms). However, the results suggest that inflation improves profitability for flexible firms, while profitability remains constant for sticky-price firms. A positive inflation shock increases ROA by 2% and EBIT margin by 20% for flexible firms, while on net ROA and EBIT margin for sticky-price firms remain unchanged.

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<sup>5</sup>I use the detailed version of the price-stickiness measure in the empirical analysis. It is presented here at the Fama-French 12 industry level for confidentiality reasons.

I rerun the profitability, leverage, and investment regressions dividing the inflation shock term and price stickiness interaction term into positive and negative categories. The goal is to see whether the effects of inflation are symmetric for positive and negative inflation, or whether there are differences between the two directions.

Changes in profitability, leverage, and investment are largely driven by deflation shocks. In Table 1.4, profitability declines following a negative inflation shock. ROA falls 3%, while EBIT margins drops 30% for flexible firms. Interestingly, deflation shocks affect sticky-price firms less. The net effect of a 1% deflation shock on a sticky-price firm is to increase ROA by about 0.7% and EBIT margin by 7%. This suggests that remaining at higher prices may be beneficial to sticky-price firms.

### 1.5.2 Leverage

The model predicts that inflation should reduce leverage in the short term. The intuition is that inflation lowers the current leverage ratio, and the endogenous adjustment costs introduced by the changing price of debt keep the leverage ratio from returning to its original value. Differences between the two sectors in the model are small. Firms with high inflation sensitivity are also more profitable when inflation is high, affecting the probability of default and potentially affecting the price of debt and its slope across next period's leverage. This in turn would change the adjustment costs associated with debt along with the marginal benefits of issuing debt.

Table 1.5 shows the results of regressing changes in book and market leverage on inflation shocks, price stickiness, and their interaction. In the data, inflation reduces leverage, consistent with the model. An increase in unexpected inflation of 1% lowers book leverage 1.1 percentage points and market leverage 2.6 percentage points. The effect is less pronounced for sticky-price firms: book leverage hardly changes, while market leverage only declines 0.6%.

In Table 1.6, I consider the effects of inflation and deflation separately on book and market leverage. Here again I find that the direction of inflation matters. Changes in book leverage are driven primarily by deflation shocks. A 1% deflation shock increases

book leverage for flexible firms by 1.8% and leaves sticky-price firms unchanged. For market leverage, both positive and negative inflation shocks appear to matter, though deflation shocks are stronger. A deflation shock raises market leverage 2.7% for flexible firms, but only 0.4% for sticky-price firms. Positive inflation shocks are also associated with a reduction in market leverage of about 2.5% for flexible firms, while the interaction term is not significant at the 10% level.

### 1.5.3 Investment

Table 1.7 shows the effects of inflation and inflation sensitivity on the investment/assets ratio across various specifications. Inflation increases investment as a consequence of decreasing debt overhang, consistent with the model. Furthermore, firms with flexible prices see a larger increase in investment. Investment is lower for sticky-price firms, and the coefficients suggest a completely inflexible firm would see almost no change at all in investment rates. In terms of magnitude, a fully flexible firm increases investment 0.7 percentage points per year in response to a 1 percentage point rise in the inflation rate. A firm with completely sticky prices decreases investment 0.2 percentage points. However, the results are not as statistically significant as the leverage results.

In Table 1.8, I report the same regressions breaking inflation and the interaction into positive and negative components. Here the results are stronger, and consistent with the previous results documenting a larger effect from deflation. A deflation shock reduces investment for flexible firms by 0.7-1.5%. For sticky-price firms, investment does not drop as far, and in fact the results suggest investment rises by 0.2-0.5%. This is consistent with the fact that profitability is also greater for sticky-price firms following deflation shocks.

## 1.6 Other empirical results

This section contains the results of other tests of the model's predictions on the effects of inflation and price stickiness on investment and leverage across various debt maturities, as well as exploring the debt overhang channel in more detail. It also discusses robustness tests of the main results.

### 1.6.1 Firm leverage, investment, and debt maturity

These results suggest that debt maturity has an important role to play in the firm's decisions. An interesting extension is to examine how the firm's investment and leverage decisions depend on maturity, in addition to price stickiness and inflation shocks. I divide firms into terciles by the average maturity of their debt, as measured by the ratio of debt in current liabilities to debt in long-term liabilities. There is some evidence of differences across the terciles. Firms with higher maturity react more strongly to inflation shocks. In particular, when I separate inflation shocks into positive and negative categories, I find that for high maturity firms leverage rises more and investment fall more than for low maturity firms. There is also a cross-sectional component coming from price stickiness. Consistent with the previous results, sticky price firms react less, while flexible firms adjust more. As Table 1.9 shows, deflation shocks affect book leverage more when firms have greater debt maturity. A 1% deflation shock raises book leverage 1.5% for short maturity firms, while leverage rises 2.0% for long maturity firms. Again, I find that deflation shocks affect flexible firms more, with sticky-price firms showing smaller changes in book leverage.

Table 1.10 reports results for investment across maturity terciles. High maturity firms reduce investment more than low maturity firms following a deflation shock. Investment falls 0.5% for low maturity firms versus 2.0% for high maturity firms. Flexible firms also reduce leverage more than sticky price firms.

### 1.6.2 Investment across leverage quantiles

Another implication of the model is that the debt overhang channel should be stronger for firms with higher debt levels. The further debt falls (mechanically induced by the inflation shock), the greater the reduction in debt overhang. To test this, I divide the sample into leverage terciles and run regressions of the investment rate on positive and negative inflation and price stickiness across the terciles. I find that in fact there are differences across firms in the three quantiles. Investment falls more for high leverage firms following a deflation shock, consistent with the debt overhang story. It also falls less for sticky price firms than for flexible firms. Table 1.11 reports the regression results. Flexible firms with high leverage see investment fall 2.0% following a 1% deflation shock. For low leverage firms, the change in investment in response to deflation is smaller (0.8%) and not statistically significant.

### 1.6.3 Robustness tests

The main leverage and investment results for the US data are robust to a variety of specifications. The results are also robust to using actual PCE inflation, as well as the difference between actual PCE inflation and the survey median expected inflation. Results also generally hold if I use the CPI price index instead of the PCE, for both actual and unexpected CPI inflation.

Results are also robust to grouping firms into quantiles by price stickiness and interacting quantile dummy variables with inflation and the inflation-price stickiness interaction term. In those regressions, I find the same patterns described above.

## 1.7 Conclusion

This paper considers the effects of nominal frictions in debt and profitability on firm investment and leverage. I jointly model the investment and debt issuance decisions for firms, taking into account the fact that inflation affects the real value of nominal



debt and sticky prices affect profitability. The model shows that these frictions lead to a positive inflation to lowering firm leverage and increasing investment, and that flexible and sticky-price firms react differently. In the data, I find that firms vary significantly in their exposure to nominal frictions. I also document a link between inflation and price stickiness and how firms invest and issue debt, as well as their profitability. Inflation raises profitability and investment rates and reduces debt issuance (and vice versa for deflation). These effects are less pronounced for sticky price firms. I find that deflation shocks in particular are the source of these changes. I also document cross-sectional patterns in investment and leverage changes across firms of varying debt maturity, and find that investment is more connected to inflation for firms with higher leverage, consistent with the debt overhang story.

## 1.8 Tables and figures

Figure 1.1: Impulse response functions for an inflation shock

This figure shows impulse response functions for several variables of interest in response to a 1 standard deviation deflationary shock. One industry sees profit fall, and investment falls accordingly. Leverage rises for both industries, with differences due to sticky prices.

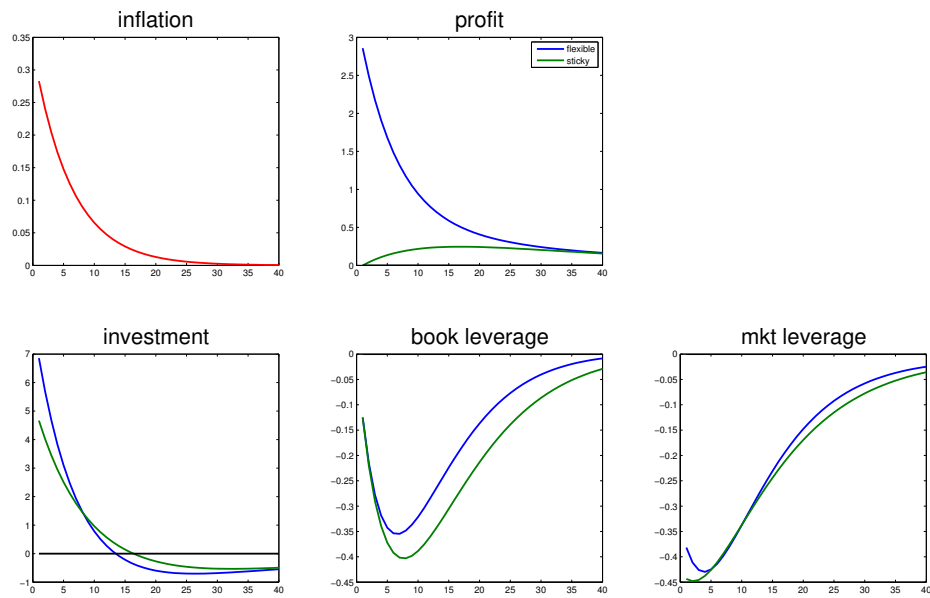


Figure 1.2: Impulse response functions for inflation and deflation shocks

This figure shows impulse response functions for several variables of interest in response to both a positive and negative 1 standard deviation shock to inflation when real profitability reacts asymmetrically to inflation. For a positive inflation shock, shown in the top row, there are no differences across industries, and inflation reduces leverage and increases investment. For a negative inflation shock, shown in the second row, profitability falls for one sector, leading to a larger decline in investment. Leverage increases, with cross-sectional differences between the two sectors.

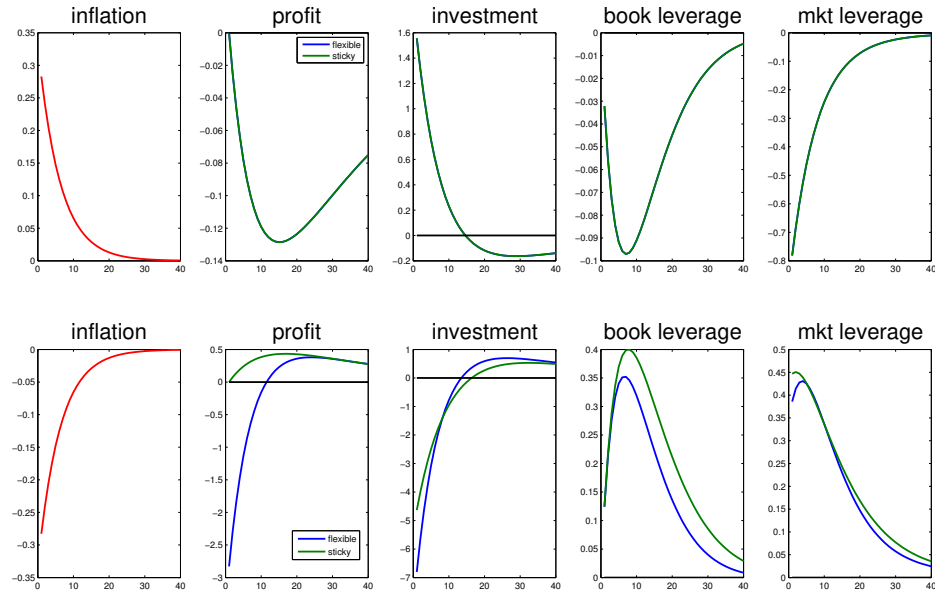


Figure 1.3: Actual and unexpected inflation rates

This figure plots actual and unexpected inflation rates for the time period 1975-2015. I use two measures of inflation, CPI from the BLS and PCE from the BEA. For unexpected inflation, I use two methods, 1) the difference between actual inflation and the median estimate of expected inflation from the Survey of Professional Forecasters, and 2) the residuals from a VAR of inflation, real GDP growth, and interest rates. SPF estimates are available starting in 1981 for CPI and 2007 for PCE. Gray bars represent NBER recession periods.

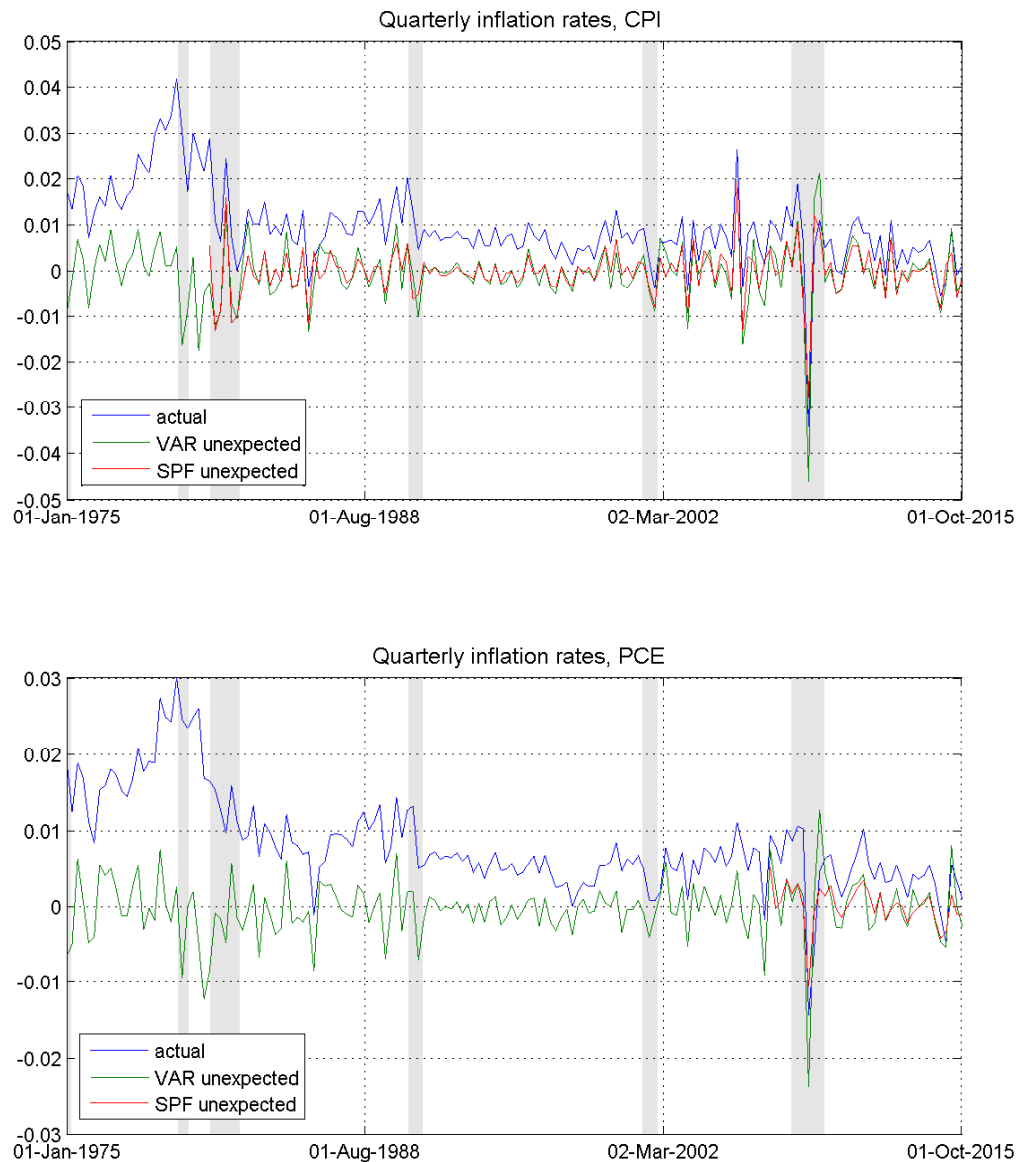


Table 1.1: Parameter values used in model calibration

This table provides the parameter values used in calibrating the model. They are largely taken from Gomes et al. (2016), and are either standard or set to match various moments including average leverage, debt maturity, interest payments, default rates, and default costs. The parameter values for profit sensitivity reflect both the symmetric and asymmetric specifications.

parameter	symbol	value
capital	$\alpha$	0.36
tax rate	$\tau$	0.4
depreciation	$\delta$	0.025
risk aversion	$\sigma$	1.0
labor supply elasticity	$\theta$	0.63
discount factor	$\beta$	0.99
1/maturity	$\lambda$	0.06
coupon rate	$c$	0.02
default cost	$\xi$	0.38
inv. adj. cost	$\eta$	5.0
$\varphi(z)$ pdf	$n_1$	0.67
profit sensitivity	$m_P$	10 / 0
	$m_N$	10 / 10

Table 1.2: Summary statistics for price stickiness measure

This table shows summary statistics for the price stickiness measure for major industries. The price stickiness measure is the average at the 6-digit NAICS industry level of the firm-level measure from Weber (2016). That paper uses price change frequency which ranges from 0 to 1. I convert into price stickiness by subtracting the frequency from 1, so that a firm with a price stickiness measure of 0 has completely flexible prices, while 1 indicates completely sticky prices. This table shows summary statistics for the price stickiness measure across the Fama-French 12 industries, excluding finance. Generally, commodities and retail have the lowest price stickiness, while services and some manufacturing have the highest.

Fama-French Industry	mean	sd	min	p25	median	p75	max
Consumer Nondurables	0.77	0.11	0.43	0.76	0.80	0.84	0.95
Consumer Durables	0.82	0.05	0.72	0.79	0.83	0.86	0.91
Manufacturing	0.81	0.10	0.30	0.78	0.83	0.87	0.96
Oil, Gas, & Coal	0.31	0.28	0.12	0.12	0.12	0.31	0.89
Chemicals	0.74	0.10	0.36	0.68	0.78	0.82	0.84
Business Equipment	0.88	0.05	0.76	0.83	0.90	0.91	0.96
Telephone & Television	0.73	0.13	0.41	0.64	0.75	0.86	0.91
Wholesale, Retail & Services	0.71	0.14	0.27	0.69	0.72	0.80	0.89
Healthcare, Med Equip, & Drugs	0.83	0.04	0.78	0.78	0.84	0.86	0.93
Other	0.74	0.17	0.17	0.61	0.77	0.84	0.93
Total	0.77	0.20	0.12	0.77	0.83	0.88	0.96

Table 1.3: Panel regressions of profitability on inflation, price stickiness, and interaction

This table presents the results of regressions of two measures of profitability, return on assets and EBIT margin, on inflation, price stickiness, and their interaction, with and without controls. ROA is operating income after depreciation divided by total assets. EBIT margin is earnings before interest and taxes divided by total sales. Inflation is the difference between the actual and expected growth rates of the BEA Personal Consumption Expenditures price index. The expected inflation rate comes from a VAR using inflation, GDP growth, and interest rates. Price stickiness (PS) is one minus the average frequency of price changes from BLS PPI data at the NAICS 6-digit industry level. Controls are the market-to-book ratio (MB), size, lagged profitability (ROA), debt maturity, interest rate, lagged investment (inv), and capital intensity (capint). All controls are lagged one quarter. Quarterly Compustat data for the period 1975-2015. Variables are winsorized at 1% where appropriate. Standard errors are clustered at the firm and quarter level.

	ROA		EBIT margin	
inflation	1.237 (1.29)	2.186*** (2.71)	14.55* (1.71)	20.43** (2.12)
PS	-0.0115*** (-3.67)	0.00361** (2.52)	-0.296*** (-9.15)	-0.0489 (-1.27)
interaction	-1.618* (-1.84)	-2.666*** (-2.67)	-19.15* (-1.89)	-24.26** (-2.00)
MB		0.000437** (2.33)		-0.0342*** (-8.23)
size		0.00154*** (12.61)		0.0150*** (7.79)
ROA		0.741*** (55.53)		5.395*** (20.32)
maturity		0.000317 (0.73)		-0.00528 (-0.40)
interest		-0.0268*** (-5.19)		-0.462*** (-5.15)
inv		-0.0188** (-2.51)		0.0302 (0.26)
capint		0.00427*** (4.04)		0.239*** (8.15)
FE	None	Y,Ind	None	Y,Ind
N	247412	145771	224300	127403
Adj. R2	0.001	0.661	0.010	0.350

clustered at firm and quarter level

\*  $p < 0.10$  , \*\*  $p < 0.05$  , \*\*\*  $p < 0.01$

Table 1.4: Panel regressions of profitability on positive and negative inflation, price stickiness, and interaction

This table presents the results of regressions of two measures of profitability, return on assets and EBIT margin, on inflation, price stickiness, and their interaction, with and without controls. ROA is operating income after depreciation divided by total assets. EBIT margin is earnings before interest and taxes divided by total sales. Inflation is the difference between the actual and expected growth rates of the BEA Personal Consumption Expenditures price index. The expected inflation rate comes from a VAR using inflation, GDP growth, and interest rates. Price stickiness (PS) is one minus the average frequency of price changes from BLS PPI data at the NAICS 6-digit industry level. Inflation and the inflation-PS interaction are separated into positive (+) and negative (-) components based on whether the inflation shock for the period is positive or negative. Controls are the market-to-book ratio (MB), size, lagged profitability (ROA), debt maturity, interest rate, lagged investment (inv), and capital intensity (capint). All controls are lagged one quarter. Quarterly Compustat data for the period 1975-2015. Variables are winsorized at 1% where appropriate. Standard errors are clustered at the firm and quarter level.

	ROA		EBIT margin	
inflation, +	-0.563 (-0.59)	0.796 (1.03)	-3.442 (-0.35)	0.717 (0.07)
inflation, -	2.288* (1.74)	3.025** (2.57)	25.27** (2.24)	32.31** (2.50)
PS	-0.0152*** (-3.81)	0.000260 (0.09)	-0.344*** (-8.29)	-0.0975* (-1.91)
interaction, +	0.294 (0.32)	-0.955 (-0.97)	5.847 (0.53)	0.524 (0.04)
interaction, -	-2.734** (-2.35)	-3.707** (-2.56)	-33.99*** (-2.73)	-39.25** (-2.47)
MB		0.000440** (2.35)		-0.0341*** (-8.23)
size		0.00154*** (12.63)		0.0151*** (7.83)
ROA		0.741*** (55.51)		5.394*** (20.30)
maturity		0.000338 (0.78)		-0.00496 (-0.37)
interest		-0.0269*** (-5.21)		-0.463*** (-5.17)
inv		-0.0187** (-2.48)		0.0305 (0.26)
capint		0.00425*** (4.05)		0.239*** (8.14)
FE	None	Y,Ind	None	Y,Ind
N	247412	145771	224300	127403
Adj. R2	0.002	0.661	0.010	0.351

clustered at firm and quarter level

\*  $p < 0.10$  , \*\*  $p < 0.05$  , \*\*\*  $p < 0.01$



Table 1.5: Panel regressions of leverage on inflation, price stickiness, and interaction

This table presents the results of regressions of leverage ratios on inflation, price stickiness, and their interaction. Book leverage is total debt over total assets, with and without controls. Market leverage is total market debt (market prices from Lehman, FISD, and TRACE) divided by market value of debt plus equity. Change in leverage is the quarter over quarter difference in the leverage ratio. Inflation is the difference between the actual and expected growth rates of the BEA Personal Consumption Expenditures price index. The expected inflation rate comes from a VAR using inflation, GDP growth, and interest rates. Price stickiness (PS) is one minus the average frequency of price changes from BLS PPI data at the NAICS 6-digit industry level. Controls are the market-to-book ratio (MB), size, lagged profitability (ROA), debt maturity, interest rate, lagged investment (inv), and capital intensity (capint). All controls are lagged one quarter. Quarterly Compustat data for the period 1975-2015. Variables are winsorized at 1% where appropriate. Standard errors are clustered at the firm and quarter level.

	Change in book leverage		Change in market leverage	
inflation	-1.283*** (-3.30)	-1.145** (-2.56)	-2.546*** (-3.15)	-2.624*** (-3.86)
PS	-0.00430*** (-4.07)	0.000286 (0.21)	-0.00362 (-1.38)	-0.000246 (-0.10)
interaction	1.230*** (3.23)	1.130** (2.35)	2.172*** (3.32)	2.003*** (3.15)
MB		-0.00347*** (-10.27)		0.00267*** (4.70)
size		0.000705*** (7.01)		0.00203*** (6.44)
ROA		-0.0253** (-2.53)		-0.0865*** (-3.68)
maturity		-0.00575*** (-6.09)		-0.00841*** (-3.09)
interest		0.115*** (8.66)		0.235*** (4.79)
inv		0.0708*** (8.18)		0.105*** (5.19)
capint		-0.00173 (-1.50)		-0.000628 (-0.24)
FE	None	Y,Ind	None	Y,Ind
N	240395	146887	31798	27950
Adj. R2	0.001	0.014	0.008	0.044

clustered at firm and quarter level

\*  $p < 0.10$  , \*\*  $p < 0.05$  , \*\*\*  $p < 0.01$

Table 1.6: Panel regressions of leverage on positive and negative inflation, price stickiness, and interaction

This table presents the results of regressions of leverage ratios on inflation, price stickiness, and their interaction. Book leverage is total debt over total assets, with and without controls. Market leverage is total market debt (market prices from Lehman, FISD, and TRACE) divided by market value of debt plus equity. Change in leverage is the quarter over quarter difference in the leverage ratio. Inflation is the difference between the actual and expected growth rates of the BEA Personal Consumption Expenditures price index. The expected inflation rate comes from a VAR using inflation, GDP growth, and interest rates. Price stickiness (PS) is one minus the average frequency of price changes from BLS PPI data at the NAICS 6-digit industry level. Inflation and the inflation-PS interaction are separated into positive (+) and negative (-) components based on whether the inflation shock for the period is positive or negative. Controls are the market-to-book ratio (MB), size, lagged profitability (ROA), debt maturity, interest rate, lagged investment (inv), and capital intensity (capint). All controls are lagged one quarter. Quarterly Compustat data for the period 1975-2015. Variables are winsorized at 1% where appropriate. Standard errors are clustered at the firm and quarter level.

	Change in book leverage		Change in market leverage	
inflation, +	-0.706 (-1.24)	-0.0524 (-0.10)	-1.640 (-1.43)	-2.498** (-2.06)
inflation, -	-1.626*** (-3.07)	-1.801*** (-3.67)	-3.123*** (-3.02)	-2.722*** (-3.02)
PS	-0.00355** (-2.14)	0.00265 (1.48)	-0.00208 (-0.67)	0.000789 (0.27)
interaction, +	0.858 (1.33)	-0.0790 (-0.12)	1.429 (1.36)	1.509 (1.45)
interaction, -	1.451*** (2.68)	1.869*** (3.92)	2.656*** (4.13)	2.298*** (3.51)
MB		-0.00347*** (-10.27)		0.00267*** (4.70)
size		0.000702*** (7.00)		0.00202*** (6.40)
ROA		-0.0252** (-2.53)		-0.0860*** (-3.66)
maturity		-0.00577*** (-6.09)		-0.00843*** (-3.09)
interest		0.115*** (8.67)		0.235*** (4.78)
inv		0.0708*** (8.19)		0.105*** (5.20)
capint		-0.00172 (-1.49)		-0.000644 (-0.25)
FE	None	Y,Ind	None	Y,Ind
N	240395	146887	31798	27950
Adj. R2	0.001	0.014	0.008	0.044

clustered at firm and quarter level

\*  $p < 0.10$  , \*\*  $p < 0.05$  , \*\*\*  $p < 0.01$

Table 1.7: Panel regressions of investment on inflation, price stickiness, and interaction

This table presents the results of regressions of investment rates on PCE inflation, price stickiness, and their interaction, with and without controls. Investment 1 is the next four quarters' average real change in gross PP&E, divided by the prior quarter's ending assets. Investment 2 is the next four quarters' average capital expenditures less sales of PP&E, divided by the prior quarter's ending assets. Inflation is the difference between the actual and expected growth rates of the BEA Personal Consumption Expenditures price index. The expected inflation rate comes from a VAR using inflation, GDP growth, and interest rates. Price stickiness (PS) is one minus the average frequency of price changes from BLS PPI data at the NAICS 6-digit industry level. Controls are the market-to-book ratio (MB), size, lagged profitability (ROA), debt maturity, interest rate, lagged investment (inv), and capital intensity (capint). All controls are lagged one quarter. Quarterly Compustat data for the period 1975-2015. Variables are winsorized at 1% where appropriate. Standard errors are clustered at the firm and quarter level.

	Investment 1 ( $\Delta PPE$ )		Investment 2 (Capex)	
inflation	0.397 (0.92)	0.656* (1.67)	0.108 (0.37)	0.282 (1.21)
PS	-0.0417*** (-15.41)	-0.0108*** (-3.92)	-0.0428*** (-19.54)	-0.0116*** (-5.70)
interaction	-0.614 (-1.40)	-0.890* (-1.78)	-0.221 (-0.72)	-0.350 (-1.20)
MB		0.00419*** (19.33)		0.00292*** (18.84)
size		-0.00156*** (-11.11)		-0.000787*** (-7.67)
ROA		0.0669*** (13.71)		0.0419*** (11.81)
maturity		-0.00341*** (-6.05)		-0.000909** (-2.16)
interest		0.0322*** (6.92)		0.0151*** (5.57)
inv		0.274*** (27.72)		0.210*** (29.08)
capint		0.0150*** (10.12)		0.0262*** (23.15)
FE	None	Y,Ind	None	Y,Ind
N	190339	129994	196201	122619
Adj. R2	0.066	0.231	0.140	0.412

clustered at firm and quarter level

\*  $p < 0.10$  , \*\*  $p < 0.05$  , \*\*\*  $p < 0.01$

Table 1.8: Panel regressions of investment on positive and negative inflation, price stickiness, and interaction

This table presents the results of regressions of investment rates on PCE inflation, price stickiness, and their interaction, with and without controls. Investment 1 is the next four quarters' average real change in gross PP&E, divided by the prior quarter's ending assets. Investment 2 is the next four quarters' average capital expenditures less sales of PP&E, divided by the prior quarter's ending assets. Inflation is the difference between the actual and expected growth rates of the BEA Personal Consumption Expenditures price index. The expected inflation rate comes from a VAR using inflation, GDP growth, and interest rates. Price stickiness (PS) is one minus the average frequency of price changes from BLS PPI data at the NAICS 6-digit industry level. Inflation and the inflation-PS interaction are separated into positive (+) and negative (-) components based on whether the inflation shock for the period is positive or negative. Controls are the market-to-book ratio (MB), size, lagged profitability (ROA), debt maturity, interest rate, lagged investment (inv), and capital intensity (capint). All controls are lagged one quarter. Quarterly Compustat data for the period 1975-2015. Variables are winsorized at 1% where appropriate. Standard errors are clustered at the firm and quarter level.

	Investment 1 ( $\Delta PPE$ )		Investment 2 (Capex)	
inflation, +	-1.351*	-0.849	-0.871	-0.527
	(-1.85)	(-1.53)	(-1.61)	(-1.45)
inflation, -	1.339***	1.478***	0.625**	0.713***
	(3.15)	(3.97)	(2.41)	(3.76)
PS	-0.0440***	-0.0144***	-0.0436***	-0.0134***
	(-13.86)	(-4.65)	(-17.86)	(-6.05)
interaction, +	0.611	1.023	0.214	0.653
	(0.68)	(1.38)	(0.31)	(1.33)
interaction, -	-1.270**	-1.937***	-0.443	-0.889***
	(-2.47)	(-4.14)	(-1.22)	(-3.77)
MB		0.00420***		0.00292***
		(19.41)		(18.89)
size		-0.00156***		-0.000786***
		(-11.09)		(-7.66)
ROA		0.0669***		0.0419***
		(13.71)		(11.79)
maturity		-0.00339***		-0.000900**
		(-6.03)		(-2.13)
interest		0.0321***		0.0150***
		(6.89)		(5.54)
inv		0.274***		0.210***
		(27.79)		(29.07)
capint		0.0150***		0.0262***
		(10.14)		(23.22)
FE	None	Y,Ind	None	Y,Ind
N	190339	129994	196201	122619
Adj. R2	0.069	0.232	0.143	0.412

clustered at firm and quarter level

\*  $p < 0.10$  , \*\*  $p < 0.05$  , \*\*\*  $p < 0.01$

Table 1.9: Panel regressions of book leverage on inflation, price stickiness, and interaction across maturity terciles

This table presents the results of regressions of changes in book leverage on inflation, price stickiness, and their interaction across debt maturity terciles, with and without controls. Book leverage is total debt over total assets. Inflation is the difference between the actual and expected growth rates of the BEA Personal Consumption Expenditures price index. The expected inflation rate comes from a VAR using inflation, GDP growth, and interest rates. Price stickiness (PS) is one minus the average frequency of price changes from BLS PPI data at the NAICS 6-digit industry level. Inflation and the inflation-PS interaction are separated into positive (+) and negative (-) components based on whether the inflation shock for the period is positive or negative. Controls are the market-to-book ratio (MB), size, lagged profitability (ROA), debt maturity, interest rate, lagged investment (inv), and capital intensity (capint). All controls are lagged one quarter. Quarterly Compustat data for the period 1975-2015. Variables are winsorized at 1% where appropriate. Standard errors are clustered at the firm and quarter level.

	Short maturity (Tercile 1)		Medium maturity (Tercile 2)		Long maturity (Tercile 3)	
inflation, +	-2.029*	-0.769	-0.657	-0.326	-0.164	0.313
	(-1.90)	(-0.86)	(-0.79)	(-0.48)	(-0.28)	(0.68)
inflation, -	-1.400**	-1.563***	-1.601**	-1.537**	-1.987***	-1.965***
	(-2.39)	(-3.13)	(-2.29)	(-2.53)	(-3.55)	(-4.02)
PS	-0.00582*	0.00415	-0.00349	0.000234	0.00142	0.00394
	(-1.86)	(1.16)	(-1.54)	(0.09)	(0.57)	(1.58)
interaction, +	2.576**	0.845	0.758	0.204	-0.101	-0.550
	(2.09)	(0.76)	(0.74)	(0.24)	(-0.13)	(-0.91)
interaction, -	1.009	1.593***	1.417**	1.499**	1.925***	2.148***
	(1.64)	(3.02)	(2.11)	(2.54)	(3.42)	(4.22)
controls	N	Y	N	Y	N	Y
FE	None	Y,Ind	None	Y,Ind	None	Y,Ind
N	66168	41339	68239	52112	67663	51819
Adj. R2	0.001	0.020	0.001	0.017	0.001	0.027

clustered at firm and quarter level, \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 1.10: Panel regressions of investment on inflation and price stickiness across maturity terciles

This table presents the results of regressions of investment rates on inflation, price stickiness, and their interaction across debt maturity terciles, with and without controls. Investment is the next four quarters' average real change in gross PP&E, divided by the prior quarter's ending assets. Inflation is the difference between the actual and expected growth rates of the BEA Personal Consumption Expenditures price index. The expected inflation rate comes from a VAR using inflation, GDP growth, and interest rates. Price stickiness (PS) is one minus the average frequency of price changes from BLS PPI data at the NAICS 6-digit industry level. Inflation and the inflation-PS interaction are separated into positive (+) and negative (-) components based on whether the inflation shock for the period is positive or negative. Controls are the market-to-book ratio (MB), size, lagged profitability (ROA), debt maturity, interest rate, lagged investment (inv), and capital intensity (capint). All controls are lagged one quarter. Quarterly Compustat data for the period 1975-2015. Variables are winsorized at 1% where appropriate. Standard errors are clustered at the firm and quarter level.

	Short maturity (Tercile 1)		Medium maturity (Tercile 2)		Long maturity (Tercile 3)	
inflation, +	-1.019*	-0.645	-0.913	-0.422	-1.783*	-0.912
	(-1.95)	(-1.26)	(-1.09)	(-0.64)	(-1.69)	(-1.26)
inflation, -	0.712***	0.505*	0.808*	0.708*	1.940***	1.984***
	(2.76)	(1.70)	(1.73)	(1.82)	(3.37)	(4.56)
PS	-0.0217***	-0.00454	-0.0327***	-0.00791*	-0.0521***	-0.0194***
	(-6.28)	(-1.18)	(-9.04)	(-1.68)	(-12.44)	(-4.82)
interaction, +	0.204	0.812	0.166	0.498	1.134	1.033
	(0.33)	(1.27)	(0.17)	(0.56)	(0.86)	(1.04)
interaction, -	-0.393	-0.648*	-0.734	-0.968**	-2.106***	-2.678***
	(-1.25)	(-1.84)	(-1.41)	(-2.06)	(-2.97)	(-4.61)
controls	N	Y	N	Y	N	Y
FE	None	Y,Ind	None	Y,Ind	None	Y,Ind
N	50144	35988	54339	46302	54984	46293
Adj. R2	0.017	0.192	0.036	0.207	0.101	0.256

clustered at firm and quarter level, \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

Table 1.11: Panel regressions of investment on inflation, price stickiness, and interaction across leverage terciles

This table presents the results of regressions of investment rates on inflation, price stickiness, and their interaction across book leverage terciles, with and without controls. Investment is the next four quarters' average real change in gross PP&E, divided by the prior quarter's ending assets. Inflation is the difference between the actual and expected growth rates of the BEA Personal Consumption Expenditures price index. The expected inflation rate comes from a VAR using inflation, GDP growth, and interest rates. Price stickiness (PS) is one minus the average frequency of price changes from BLS PPI data at the NAICS 6-digit industry level. Inflation and the inflation-PS interaction are separated into positive (+) and negative (-) components based on whether the inflation shock for the period is positive or negative. Controls are the market-to-book ratio (MB), size, lagged profitability (ROA), debt maturity, interest rate, lagged investment (inv), and capital intensity (capint). All controls are lagged one quarter. Quarterly Compustat data for the period 1975-2015. Variables are winsorized at 1% where appropriate. Standard errors are clustered at the firm and quarter level.

	Low leverage (Tercile 1)		Medium leverage (Tercile 2)		High leverage (Tercile 3)	
inflation, +	-0.0395 (-0.05)	0.00863 (0.01)	-0.615 (-0.67)	-0.556 (-0.92)	-2.447*** (-2.72)	-1.059 (-1.51)
inflation, -	0.422 (0.88)	0.800 (1.26)	0.742 (1.44)	0.784** (2.02)	2.249*** (4.46)	2.052*** (5.23)
PS	-0.0420*** (-8.71)	-0.0103* (-1.93)	-0.0436*** (-11.42)	-0.0150*** (-3.67)	-0.0457*** (-11.47)	-0.0145*** (-3.83)
interaction, +	-1.079 (-1.22)	0.0360 (0.04)	-0.184 (-0.16)	0.604 (0.76)	2.040* (1.92)	1.311 (1.36)
interaction, -	-0.0660 (-0.11)	-1.055 (-1.38)	-0.602 (-0.98)	-1.049** (-2.14)	-2.494*** (-4.09)	-2.732*** (-5.38)
controls	N	Y	N	Y	N	Y
FE	None	Y,Ind	None	Y,Ind	None	Y,Ind
N	64389	21279	65216	54764	60733	53948
Adj. R2	0.060	0.273	0.081	0.264	0.062	0.216

clustered at firm and quarter level, \*  $p < 0.10$ , \*\*  $p < 0.05$ , \*\*\*  $p < 0.01$

## Data appendix

This appendix provides a list of variables used and their derivation from Compustat data. All quantities are converted to real values by dividing by the quarterly PCE price level. Variables are winsorized at 1% where appropriate.

The sample is quarterly from 1975-2015. I restrict observations to firms on the AMEX, NYSE, and Nasdaq exchanges. I drop financial firms and utilities. I only consider firms with more than \$5 million in real assets.

*Book leverage:* Total book debt (dltt+dlc) divided by total assets (at)

*Market leverage:* Book debt (dltt+dlc) adjusted for market prices from Lehman, FISD, and TRACE, divided by total firm market value (market value of equity plus market value of debt)

*Investment 1:* Change in real gross PP&E (ppeg less prior quarter ppeg, equivalent to ppent less prior quarter ppent plus depr). Investment is averaged over the following four quarters to smooth lumpiness.

*Investment 2:* Capital expenditures (capx) less sale of property, plant and equipment (sppe). sppe set to zero if missing. Investment is averaged over the following four quarters to smooth lumpiness.

*Size:* log of firm value

*ROA:* operating income after depreciation (oiadp) divided by total assets (at)

*EBIT margin:* earnings before interest and taxes divided by total sales (sale)

*M/B:* firm market value divided by total assets (at)

*Maturity:* Debt in current liabilities (dlc) divided by total book debt (dltt+dlc)

*Capital intensity:* Fixed assets (ppent) divided by total assets (at)

*Interest rate:* interest (xint) divided by book debt (dltt+dlc)



## Model derivations

### Detailed derivation of model first order conditions

The model is described by

$$v(\omega, A, \mu) = \max_{i, \omega'} \left\{ p(\omega') \left( \omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) - h(i) + \tau \delta \right. \\ \left. + g(i) E \left[ M' \int_Z eq(\omega', \mu', A', z') d\Phi(z') \right] \right\}$$

$$eq(\omega, A, \mu, z) = \max \left\{ 0, (1 - \tau)(R(A, \mu) - z) - ((1 - \tau)c + \lambda) \frac{\omega}{\mu} + v(\omega, A, \mu) \right\}$$

The ratio of next period capital to today's capital is  $g(i) \equiv k'/k = 1 - \delta + i$ . The investment rate including adjustment costs is  $h(i) = i + \frac{\eta}{2} (i)^2$ .

The optimal default threshold for  $z$  is

$$z^*(A, \mu) = R(A, \mu) - \left( \frac{(1 - \tau)c + \lambda}{1 - \tau} \right) \frac{\omega}{\mu} + \frac{v(\omega, A, \mu)}{1 - \tau}$$

with

$$\frac{\partial z^*(A, \mu)}{\partial \omega} = - \left( \frac{(1 - \tau)c + \lambda}{1 - \tau} \right) \frac{1}{\mu} + \frac{1}{(1 - \tau)} \frac{\partial v(\omega, A, \mu)}{\partial \omega}$$

and the price of leverage is

$$p(\omega') \omega' = E \left[ M' \left( \Phi(z^{*'}) \frac{c + \lambda + (1 - \lambda)p'}{\mu'} \omega' \right. \right. \\ \left. \left. + \int_{z^{*'}}^{z_H} \left( (1 - \tau)(R' - z') + (1 - \lambda) \frac{p' \omega'}{\mu'} + v(\omega', \mu') - \xi \right) d\Phi(z') \right) \right]$$

Differentiating the value function w.r.t.  $i$ ,

$$\begin{aligned}
0 &= p(\omega')\omega' - h'(i) + E \left[ M' \int_Z eq(\omega', \mu', A', z') d\Phi(z') \right] \\
0 &= p(\omega')\omega' - 1 - \eta i + E \left[ M' \int_Z eq(\omega', \mu', A', z') d\Phi(z') \right] \\
1 + \eta i &= p(\omega')\omega' + E \left[ M' \int_Z eq(\omega', \mu', A', z') d\Phi(z') \right]
\end{aligned}$$

The FOC equates the marginal cost of investment today,  $1 + \eta i$ , with the marginal benefit, higher profits in the next period.

Differentiating the value function w.r.t.  $\omega'$  using the rule

$$\frac{d}{d\omega'} \int_{a(\omega')}^b f(z', \omega') d\Phi(z') = \int_{a(\omega')}^b \frac{\partial f(z', \omega')}{\partial \omega'} d\Phi(z') - f(a(\omega'), \omega') a'(\omega'),$$

$$0 = \frac{\partial p}{\partial \omega'} \left( \omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) + p(\omega') g(i) + g(i) \frac{\partial}{\partial \omega'} E \left[ M' \int_Z eq(\omega', \mu', A', z') d\Phi(z') \right]$$

Focusing on the last term,

$$\begin{aligned}
&\frac{\partial}{\partial \omega'} E \left[ M' \int_Z eq(\omega', \mu', A', z') d\Phi(z') \right] \\
&= E \left[ M' \frac{\partial}{\partial \omega'} \int_{z^{*'}}^{z^H} eq(\omega', \mu', A', z') d\Phi(z') \right] \\
&= E \left[ M' \left( \int_{z^{*'}}^{z^H} \frac{\partial}{\partial \omega'} eq(\omega', \mu', A', z') d\Phi(z') - eq(\omega', \mu', A', z^{*'}) \frac{\partial z^{*'}}{\partial \omega'} \right) \right] \\
&= E \left[ M' \left( \int_{z^{*'}}^{z^H} \left( -\frac{((1 - \tau)c + \lambda)}{\mu'} + \frac{\partial v(\omega', A', \mu')}{\partial \omega'} \right) d\Phi(z') - eq(\omega', \mu', A', z^{*'}) \frac{\partial z^{*'}}{\partial \omega'} \right) \right] \\
&= E \left[ M' \int_{z^{*'}}^{z^H} \left( -\frac{((1 - \tau)c + \lambda)}{\mu'} + \frac{\partial v(\omega', A', \mu')}{\partial \omega'} \right) d\Phi(z') \right] \\
&= E \left[ M' \int_{z^{*'}}^{z^H} (1 - \tau) \frac{\partial z^{*'}}{\partial \omega'} d\Phi(z') \right] \\
&= E \left[ M' (1 - \tau) \Phi(z^{*'}) \frac{\partial z^{*'}}{\partial \omega'} \right]
\end{aligned}$$

Line 3 comes from taking the partial derivative of  $eq'$  w.r.t.  $\omega'$ . In line 4, the value of  $eq'$  at  $z^{*'}$  is zero since  $z^{*'}$  is the default threshold, so the last term is zero. Line 5 substitutes in the default threshold, which is independent of  $z'$ , giving line 6.

Substituting this into the FOC, we get

$$\begin{aligned}
0 &= \frac{\partial p}{\partial \omega'} \left( \omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) + p(\omega') g(i) + g(i) E \left[ M' (1 - \tau) \Phi(z^{*'}) \frac{\partial z^{*'}}{\partial \omega'} \right] \\
p(\omega') g(i) &= - \frac{\partial p}{\partial \omega'} \left( \omega' g(i) - (1 - \lambda) \frac{\omega}{\mu} \right) - (1 - \tau) g(i) E \left[ M' \Phi(z^{*'}) \frac{\partial z^{*'}}{\partial \omega'} \right]
\end{aligned}$$

## Model solution details

I follow the model solution used in Gomes et al. (2016). I use Dynare to solve a first order perturbation approximation around the model steady state. As in Gomes et al. (2016), the solution depends on the firm's policy function for leverage. This is because, with long-term debt, the price of debt today depends on tomorrow's debt price, which is a function of tomorrow's leverage choice, which in turn depends on today's leverage choice. For simplicity, I assume that future prices are unaffected by the firm's leverage choice today, so that the firm does not need to know its policy function.

# Chapter 2

## Inflation, Welfare, and the Fisher Effect

### 2.1 Introduction

The Fisher effect has been widely recognized since Fisher’s *The Theory of Interest* (1930) and is presented as a ‘stylized fact’ to students of economics and finance.<sup>1</sup> For example, Mankiw’s *Macroeconomics* (2012) states, “According to the Fisher equation, a 1 percent increase in the rate of inflation in turn causes a 1 percent increase in the nominal interest rate. The one-for-one relation between the inflation rate and the nominal interest rate is called the Fisher Effect” (p. 111). A theoretical derivation of the Fisher effect is found in Wallace (2012) in the form of a no-arbitrage condition:

One way to do this is to (i) sell the date  $t$  good for money obtaining  $P_t$  units of money, (ii) lend the money at  $i_t$  thereby acquiring  $(1 + i_t)P_t$  units of money at date  $t + 1$ , and (iii) use that money to buy date  $t + 1$  good in the amount  $(1 + i_t)\frac{P_t}{P_{t+1}}$ . An alternative is to lend the date  $t$  good

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<sup>1</sup>We thank seminar participants at the 2012 University of Wisconsin Monetary Theory meetings, the Carlson School finance department, and the 2015 Midwest Finance Association meeting. We also thank Philip Bond, Jose Jorge, Neil Wallace, Steve Williamson, and Randy Wright for helpful comments on earlier drafts. Most of all, we thank our friends and colleagues Hendrik Hakenes (University of Bonn) and Ross Levine (Berkeley) for their many comments on various drafts of this study.

at the real rate  $r_t$ . This gives  $1 + r_t$  units of the date  $t + 1$  good. Equating the two amounts of date  $t + 1$  good, we get  $(1 + i_t) = (1 + r_t) \frac{P_{t+1}}{P_t}$ . This is often called the Fisher equation.

The point of the present study is to prove, in a general equilibrium framework, that the Fisher effect does not hold for defaultable debt. However, we need to be clear about one thing from the beginning. Our arguments are contract-theoretic, microeconomic, and hold only for defaultable debt. To make matters simple and clean, we assume an environment in which the Fisher relation holds perfectly for default-risk-free debt. Then, the interest rate on defaultable debt is shown to increase with inflation, but less than the “one-for-one” relationship predicted by Fisher.

The intuition is as follows. For simplicity, we consider two states of inflation, high and low. We impose that the nominal repayment to the investor must be the same in both states. In the high inflation states, as average inflation increases, the real value of the contract to the investor approaches zero. Also, the real value of that piece of the contract falls faster at low average inflation than it does at higher average inflation. Thus the investor recovers the required return more and more in the low inflation state, and as the value of the high-inflation state goes to zero, entirely there.

After proving this, we take the theory to the data, employing cross-section observations for 74 countries and 24 years. The predictions of our theory are strongly supported by the empirics. A key robustness check is to substitute government Treasury Bill rates for rates on corporate debt, from the same years and same countries. With government debt we obtain a one-for-one linear relationship exactly *a la* Fisher. This is consistent with the theory since our theorem only holds when default (bankruptcy) is possible. And it reduces the likelihood that our empirical results are due to some form of mis-specification. These findings have several important implications for development finance and development policy. As discussed in a companion paper (Boyd and Jalal, 2014), this is consistent with a large empirical literature in development economics suggesting that high inflation is associated with reduced real growth. We focus here on inflation and the Fisher effect.

The rest of the paper proceeds as follows. Section II presents the theory and some numerical examples. Theorem 1 proves that in this environment inflation results in deadweight losses and reduced welfare. This section next investigates the Fisher relationship and presents our second theorem: the interest rate on defaultable debt increase less than proportionally with the rate of inflation. Thus, it does not obey the Fisher relationship. Section III presents the empirics and Section IV concludes.

## 2.2 The theory

### 2.2.1 Agents and technologies

The economy lasts two periods and all agents are risk-neutral. There is a continuum of agents on  $[0,1]$  and all agents value only consumption in period 1. There are two classes of agents, lenders and entrepreneurs. Lenders are endowed in period 0 with  $E$  units of the single good and a home production technology that earns a constant gross rate of interest of  $\alpha$  which is public knowledge. Entrepreneurs are endowed with no goods but are endowed with access to an investment opportunity whose rate of return stochastically dominates  $\alpha$ . Their production technology converts the period 0 good into the period 1 good. The production opportunity is constant returns to scale, risky, and requires an investment strictly equal to  $E$ . To undertake the investment the entrepreneur must therefore borrow from a lender. The net amount available for investment is  $E$  and to keep things simple we assume that  $E$  is the only scale possible. The real rate of return on investment  $r$  is a random variable and we assume that  $r$  is uniformly distributed on the interval  $[L, U]$ , where  $L$  and  $U$  are exogenously given and  $0 \leq L < U$ .

### 2.2.2 Costly state verification

When returns are realized in period 1, they are freely observed by the entrepreneur. A lender can observe these return realizations only by paying a fixed monitoring cost

$M$ , which is a dead-weight cost. This is a costly state verification environment and it is well known that in such an environment, the optimal contract is a “standard debt contract” – one that promises a constant payment to the lender in all states in which monitoring does not occur. Hereafter, we will call this promised constant return a “gross interest payment,”  $I$ . In equilibrium, the lender receives the lesser of  $I$  and the total value of the firm  $r$  minus monitoring costs. Under this piece-wise linear return structure, the entrepreneur becomes residual claimant in non-monitoring states and thus is essentially an equity holder. Monitoring states are associated with bankruptcy and the investor has the rights of a debt holder.<sup>2</sup> We assume that agents have full ability to commit to contract terms in period zero, and further for technical reasons we assume that contracts cannot employ lotteries or other forms of extrinsic uncertainty.<sup>3</sup>

The nature of the game is that each entrepreneur seeks to maximize his expected return subject to the participation of a single lender. By assumption, entrepreneurs are in short supply and will earn all gains from trade. There is a single choice variable, the gross real interest payment  $I$ . Normalizing  $E$  to 1, the expected return of an entrepreneur is

$$\Delta = \int_I^U (r - I) f(r) dr \quad (2.2.1)$$

which represents the residual return to the project in states in which interest is paid and no monitoring occurs.

The expected return of a lender is

$$\Omega = I \int_I^U f(r) dr + \int_L^I (r - M) f(r) dr \quad (2.2.2)$$

where the first term is the expected real interest payment in non-monitoring states

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<sup>2</sup>The optimality of this arrangement has been shown previously, for example in Gale and Hellwig (1985) and Townsend (1978). In appendix 2.5.1, we provide a proof for the nominal contract’s optimality.

<sup>3</sup>See Boyd and Smith (1994) for an explanation of the technical reasons why we must prohibit “extrinsic uncertainty.” They further show that this theoretical restriction is totally irrelevant in actual applications of the model.

and the second term is the recovery of investors in monitoring states, net of monitoring costs. An equilibrium solves  $\max_I \{\Delta\}$  subject to  $\Omega \geq \alpha$ .

### 2.2.3 Inflation, nominal contracting and welfare

We introduce inflation and nominal contracting in a stylized way so as to avoid writing down and solving a monetary general equilibrium model.<sup>4</sup> We make two critical assumptions. First, agents are restricted to using nominal contracts. We know, based on experience in high inflation environments, that if inflation is sufficiently high, real contracts will emerge and will be increasingly employed if inflation persists. However, our model is not intended to deal with hyperinflation – only environments with moderate inflation where agents continue using nominal contracts. These are the environments in which the Fisher equation is of interest. The second key assumption is that, as the rate of inflation increases, the future price level becomes less predictable. Existing empirical work strongly supports this second assumption (Barnes, 1999; Barnes, Boyd and Smith, 1999; Boyd, Levine and Smith, 2001; Boyd and Jalal, 2014).

As will be seen, the combination of these two assumptions introduces uncertainty regarding future prices which complicates the contracting problem. Obviously, the problem could be eliminated by writing real contracts, a possibility that we prohibit by assumption.<sup>5</sup> We also ignore the question as to why the government would pursue inflationary policies. There are many possible explanations, an obvious one being seignorage.

In period 0, lenders and entrepreneurs enter into nominal contracts determining their payoffs in period 1. At the end of period 0, after such contracts are written, a price innovation determines the actual price level in period 1. The monetary authority chooses an expected rate of inflation but by assumption cannot precisely determine

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<sup>4</sup>That could be done, we believe, in the context of an overlapping-generations model with appropriate cash-in-advance constraints. For present purposes, however, that would add substantial complexity and little additional insight.

<sup>5</sup>In reality, writing real contracts is costly. The incentives to do so increase with the level and variability of inflation. (Boyd, Levine and Smith, 2001).



the actual future price level. In period 0, the decision period, the price level is  $P_0$ . The monetary authority chooses a mean price level for period 1 of  $P_M$ , but the actual price level in period 1,  $P_1$ , is a random variable. A simple way to model this is to assume that the probability density function of  $P_1$  places all its mass on two points, a high price,  $P_H$ , and a low price,  $P_L$ . The probabilities of the high and low states are assumed to be equal to  $1/2$ . Therefore  $P_H \geq P_M \geq P_L$  and  $P_M = \frac{P_H + P_L}{2}$ . Further, we normalize prices so that  $P_L = 1$ . This assumption guarantees that as average inflation rises, the variability of inflation rises also.

In period 0, the entrepreneur and the investor enter into a nominal debt contract to be paid in period 1 after project returns are realized. The nominal debt payment (principal and interest) promised to the investor is  $\Lambda$ . We assume that in period 0 the average future price level  $P_M$  and the distribution function of  $P_1$  are common knowledge. We show in Appendix 2.5.1 that the optimal contract takes the standard form in each state of inflation; that is, we are still in a CSV environment. Therefore, the expected real period 1 return to the entrepreneur can now be written in two parts,

$$\Delta_N = \frac{1}{2} \int_{\Lambda}^U (r - \Lambda) f(r) dr + \frac{1}{2} \int_{\frac{\Lambda}{P}}^U \left( r - \frac{\Lambda}{P} \right) f(r) dr \quad (2.2.3)$$

reflecting the two future equi-probable price levels.

The expected real return to the lender can similarly be written in two parts,

$$\begin{aligned} \Omega_N = \frac{1}{2} \left[ \Lambda \int_{\Lambda}^U f(r) dr + \int_L^{\Lambda} (r - M) f(r) dr \right] \\ + \frac{1}{2} \left[ \frac{\Lambda}{P} \int_{\frac{\Lambda}{P}}^U f(r) dr + \int_L^{\frac{\Lambda}{P}} (r - M) f(r) dr \right] \end{aligned} \quad (2.2.4)$$

An equilibrium is defined by  $\max_{\Lambda} \{\Delta_N\}$  subject to  $\Omega_N \geq \alpha$ .

The constraint  $\Omega_N \geq \alpha$  should always hold as an equality at any interior solution. Then, the solution comes only from (2.2.4) driven to the reservation utility  $\alpha$ . This occurs because if a lender receives more than  $\alpha$ , a competing lender could offer a lower repayment to the entrepreneur and still be above its reservation utility. Thus

in equilibrium the investor's constraint binds.

This leads to several interesting results. First, expected monitoring costs are increasing in inflation. Since monitoring results in dead-weight costs, higher inflation leads to welfare loss in this environment.

**Theorem 1.** *Define expected monitoring costs as the expected amount paid in monitoring costs given the nominal contracting problem:*

$$\mu_N = \frac{1}{2} \int_L^\Lambda M f(r) dr + \frac{1}{2} \int_L^{\frac{\Lambda}{P}} M f(r) dr = \frac{M}{2(U-L)} \left( \Lambda \left( \frac{P+1}{P} \right) - 2L \right).$$

*Then  $\frac{\partial \mu_N}{\partial P} > 0 \forall P \geq 1$ . That is, expected monitoring costs increase in inflation.*

*Proof.* Taking the derivative of  $\mu_N$  with respect to  $P$  and simplifying gives that

$$\frac{\partial \mu_N}{\partial P} = \frac{M}{2(U-L)} \left( \frac{\Lambda^2}{P} \right) \left( \frac{1-P}{\Lambda(P^2+1)-(U-M)(P^2+P)} \right) > 0 \forall P \geq 1. \text{ See Appendix 2.5.2 for details.} \quad \square$$

A numerical example of this effect is shown in Table 2.1, which we discuss further below. Basically, rising inflation complicates the contracting problem and leads to more states of nature in which costly monitoring is necessary.

## 2.2.4 Inflation and the Fisher effect

Letting the Fisher repayment  $\Lambda_F$  equal the real repayment derived in 2.2 times expected inflation,  $\Lambda_F = \frac{P+1}{2}I$ , we can prove the following result.

**Theorem 2.** *Given the above environment,  $\Lambda_F(P) > \Lambda_N(P)$  for  $P > 1$  when  $(U - M)^2 \geq \frac{3025}{336} \left( \alpha(U - L) - ML + \frac{L^2}{2} \right)$*

*Proof.* We can show this is true by directly comparing the solutions for the nominal and Fisher repayment levels and showing that the relationship is satisfied given the above parameter restriction. See the appendix for details of the proof and determining the parameter restriction.  $\square$

This parameter restriction is a sufficient condition. That is, if the restriction is satisfied, then the nominal repayment  $\Lambda_N$  will be less than the Fisher repayment  $\Lambda_F$

for all possible levels of positive inflation. An example of parameters that satisfy the restriction are  $U = 10$ ,  $M = L = 0.5$ ,  $\alpha = 1.02$ . Note also that the side constraint  $L < \frac{\Lambda_N(P)}{P} < U$ , which says that the real repayment must be feasible in the high inflation state of the world, effectively imposes an upper bound on  $P$ .

### 2.2.5 Numerical examples

In Table 2.1, the first column shows solution values when  $P_M = 1$  and the mean rate of inflation is zero in which case, we have the real contracting outcome. Column 2 (3) shows solutions when average inflation is 10% (20%). The first thing to observe is that entrepreneurs fare better when unexpected inflation is low and lenders fare better when unexpected inflation is high. This is hardly surprising. The second observation is that, as per Theorem 1, when average inflation increases, expected equilibrium monitoring costs rise. Monitoring costs are a deadweight loss and therefore the expected consumption of all agents must decline (as it does, see row 10). In this example, as inflation increases from 0 to 20 percent, deadweight losses increase by about 1.3 percent. This happens because nominal contracting renders the contract rules less precise, and the effect becomes more severe as inflation rises.<sup>6</sup>

The second important result in Table 2.1 is that as inflation increases the nominal repayment  $\Lambda_N$  increases also, but less than proportionally (see row 13), e.g. the result in Theorem 1. The economic intuition for the result is actually straightforward. A standard debt contract pays a total expected return that is a weighted average; it is *partly nominal* (the gross interest return), and *partly real* (recovery in bankruptcy states). Thus, if the nominal component were to increase proportionally with

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<sup>6</sup>To further elucidate this point, consider a different two-state environment in which real returns can only be 1.0 and 1.5, both equi-probable. Also, assume that parameters are such that with no inflation it is optimal to monitor only in the low real return state – so that monitoring would occur one half the time. Next, assume that there is inflation and that the period one price level can take on two values: 1.0 and 1.5, both equi-probable. There are now four possible outcomes: low return, low inflation; low return, high inflation; high return, low inflation; and high return, high inflation. To successfully monitor in all desired cases (those in which real returns are low), requires monitoring 75 percent of the time. Monitoring only in the low return, low inflation case is inefficient. Thus, in this example, the equilibrium is one in which one third of total monitoring is unnecessary, vis-a-vis the no inflation environment.

Table 2.1: Numerical example

1	$P_H$	1.0	1.2	1.4
2	$P_M$	1.0	1.1	1.2
3	$P_L$	1.0	1.0	1.0
4	Return to entrepreneurs, low inflation	1.5504	1.4668	1.3968
5	Return to entrepreneurs, high inflation	1.5504	1.6339	1.7037
6	Return to investors, low inflation	1.0300	1.1114	1.1795
7	Return to investors, high inflation	1.0300	0.9486	0.8805
8	Monitoring cost, low inflation	0.0196	0.0218	0.0237
9	Monitoring cost, high inflation	0.0196	0.0175	0.0157
10	Expected monitoring cost	0.019627	0.019657	0.019728
11	Monitoring cost / output	0.008178	0.008190	0.008220
12	$\Lambda$	1.1421	1.2475	1.3381
13	$\Lambda/P_M$	1.1421	1.1341	1.1151

Parameter assumptions:  $L = 0.2$ ,  $U = 5$ ,  $E = 1$ ,  $M = 0.1$ ,  $\alpha = 1.03$ ,  $P_L = 1$ .

inflation, investors would necessarily be over-compensated.

## 2.3 The Fisher effect: taking theory to data

In this section we test the theoretical prediction that, as inflation increases away from zero, the nominal interest rate on defaultable debt increases less than proportionally. We employ two datasets from 74 countries over 24 years. “Lending Rate” comes from the World Development Indicators.<sup>7</sup> It is defined as the rate charged by banks on loans to prime corporate customers. There is one observation per country/year. Our second data source is an individual firm panel dataset from Compustat that includes 30,576 firms in the same countries and years. We exclude the US from the data because it dominates the Compustat sample. In robustness tests, we show that our results continue to hold exclusively for US data and when we use both US and international data.

With the firm-level data from Compustat we cannot measure the marginal cost of corporate borrowing directly, but must estimate it. All we have for each firm/year

<sup>7</sup><http://data.worldbank.org/indicator>

data point is total debt outstanding and total interest paid. The ratio of the two is an average interest rate on all of the firm's debt outstanding and is not the marginal object we wish to measure. Below, we explain how we estimate marginal debt costs with the Compustat firm data.

### 2.3.1 Estimating marginal borrowing costs with individual firm data

With the individual firm data we observe 4 objects in 2 years for each firm. Define  $Int_t$  as the interest paid in year  $t$  and  $D_t$  as the total debt outstanding in year  $t$ . (For simplicity, firm subscripts are omitted.) If either  $D_t < 0$  or  $Int_t < 0$ , we delete the observation as it is probably a reporting error. We also delete all cases with  $D_t < D_{t-1}$  since this probably indicates that no new debt was issued. If no new debt was issued, there is no new marginal rate for us to record. This procedure deletes about one half of the sample.

Define  $r_{t-1} \equiv \frac{Int_{t-1}}{D_{t-1}}$  as the historic average interest rate that is observable. Define  $r_t$  as the new marginal interest rate that we cannot observe. Now, we have to make some assumptions. If we assume that all old debt is short term and refinanced every period, the marginal rate is equal to the average rate and  $r_t = \frac{Int_t}{D_t}$ . If, on the other hand, we assume that all old debt is fixed rate and not refinanced,  $Int_t = Int_{t-1} + r_t(D_t - D_{t-1})$  and  $r_t = \frac{Int_t - Int_{t-1}}{D_t - D_{t-1}}$ , the change in interest paid, divided by the change in total debt outstanding.

Any case between the two extremes is possible. We use the second definition that no debt is refinanced in the following empirical work. However, we find similar results under the assumption that all debt is refinanced every period.

### 2.3.2 Regression results

In the regressions presented in Table 2.2, the dependent variable is one of the interest rate measures discussed above, and the explanatory variable is average annual

inflation.<sup>8</sup> There are dummy variables for year, and clustering for country or firm. We delete all observations with average inflation exceeding 60%. Such data are outside the range of economies the model is intended to describe. Finally, we delete all observations with negative interest rates or with interest rates exceeding 100%.<sup>9</sup>

In Table 2.2, Panel A, column 1, we present results when the dependent variable is Lending Rate, from the World Development Indicators. Inflation enters in both the level and in a squared term ( $Inflation^2$ ). The first term is positive and highly significant and the second is negative and highly significant. The regression is plotted in Figure 2.1(a) with 90% confidence bands. Also plotted for purposes of reference is a 45 degree line emanating from the function's intercept. It is clear that with these data, the nominal interest rate on corporate debt does not keep pace with the rate of inflation.

In Table 2.2, Panel A, column 2, we present results with the individual firm data where the dependent variable is the estimated marginal corporate debt rate on debts of all maturities. The explanatory variables are as described previously. As seen in Figure 2.1(b), the coefficient of inflation is positive and highly significant and the coefficient of inflation squared is negative and highly significant. Again, the estimated marginal corporate borrowing rate does not keep pace with the average rate of inflation. In Table 2.2, Panel A, column 3 we present results with the individual firm data when the dependent variable is the marginal interest rate on long term debt. The regression results and plot are very similar to the others.

### 2.3.3 Robustness tests

The theory predicts a non-proportional response of the nominal interest rate on defaultable debt and we find support for that in the data. However, default-risk-free

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<sup>8</sup>We have conducted robustness tests where inflation is averaged over 3 and 5 year periods, with commensurate decrease in sample size. For brevity, those results are not presented but they are completely consistent with the results in the paper.

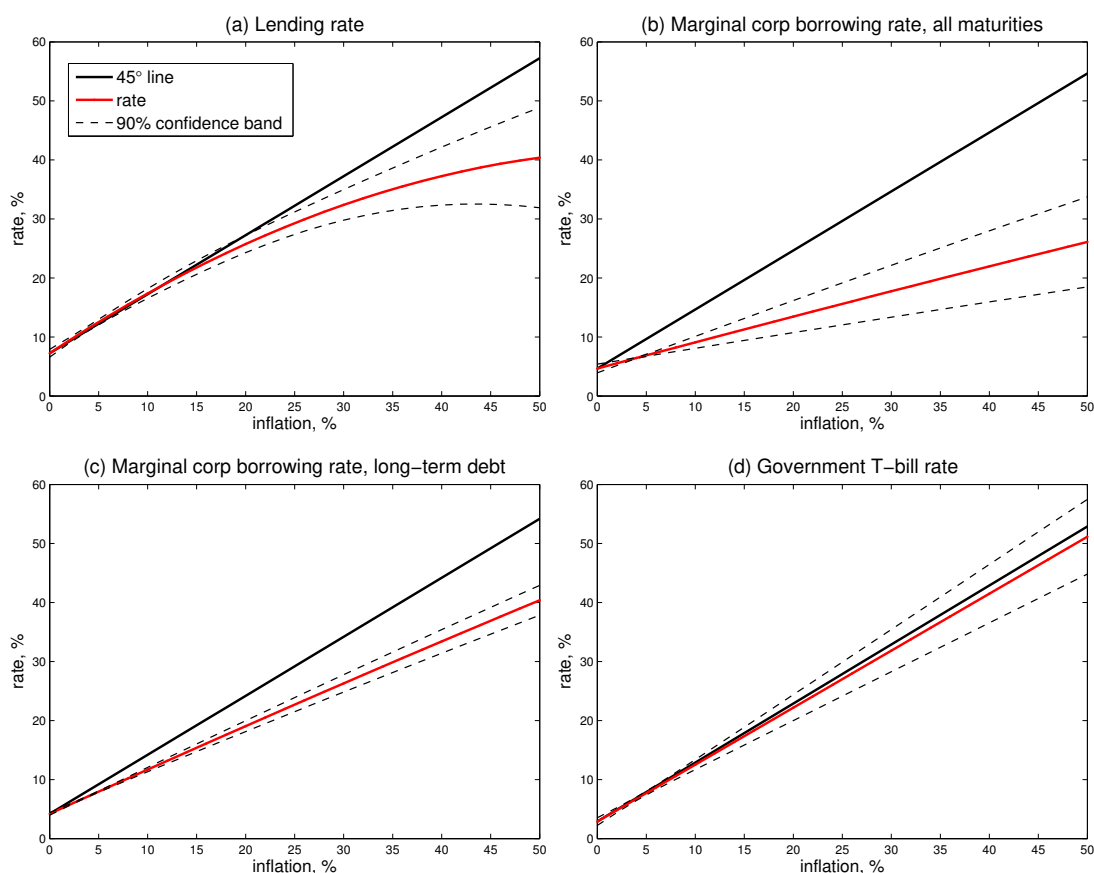
<sup>9</sup>Our results are not particularly sensitive to the choice of these thresholds.

Table 2.2: Regressing interest rates on inflation

Panel A: Corporate debt			
	(1) Lending Rate	(2) M Total Debt	(3) M Long Debt
Inflation	1.1016 (0.0856)***	0.4472 (0.0899)***	0.7564 (0.0291)***
Inflation <sup>2</sup>	-0.0088 (0.0029)***	-0.0004 (0.0001)***	-0.0007 (0.0000)***
Constant	7.2622 (0.3342)***	4.6558 (0.3781)***	4.1972 (0.1093)***
Obs.	1204	51477	35409
Adj. $R^2$	0.37	0.15	0.21
* $p < 0.05$ , ** $p < 0.01$ , *** $p < 0.001$			
Panel B: Government debt			
	(4) Govt T-bill	(5) Govt T-bill	
Inflation	1.0716 (0.1222)***	0.9654 (0.0706)***	
Inflation <sup>2</sup>	-0.0037 (0.0052)		
Constant	2.5506 (0.3557)***	2.8885 (0.3314)***	
Obs.	857	857	
Adj. $R^2$	0.59	0.58	

Dependent variables are defined at the top of the columns for each panel. In Panel A, which uses corporate debt, Lending Rate is the rate charged by banks on loans to prime customers. M Total Debt is the marginal rate of return on the total debt (long-term + short-term) of firms. M Long Debt is the average marginal rate of return on the long-term debt of firms. In Panel B, which considers government debt, Govt T-bill is the short-term rates on government securities. Inflation is the annual CPI inflation. Inflation<sup>2</sup> is the squared value of Inflation. All regressions include year dummy variables and firm or country clustering. Standard errors are in parentheses. These regressions do not include observations from the US.

Figure 2.1: Fitted interest rates plotted against inflation



This figure plots the best fit from the various regressions against the 45-degree line plotted through the regression's intercept, which is the Fisher case. It also includes 90% confidence bands. The horizontal axis represents inflation and the vertical axis is the corresponding interest rate. (a) is the lending rate from Table 2.2, Panel A, column 1. (b) is the marginal corporate borrowing rate across all maturities from Table 2.2, Panel A, column 2. (c) is the marginal corporate borrowing rate for long-term debt only from Table 2.2, Panel A, column 3. (d) is the government T-bill rate from Table 2.2, Panel B, column 5.



Table 2.3: Robustness tests on US data

	(1) AAA Corporate	(2) Tbill
Inflation	1.1077 (0.3364)***	0.9924 (0.4541)**
Inflation <sup>2</sup>	-0.0443 (0.0227)*	0.0954 (0.0635)
Constant	4.8311 (0.7556)***	0.8262 (0.8346)
Obs.	35	29
Adj. $R^2$	0.42	0.52

\*  $p < 0.05$  , \*\*  $p < 0.01$  , \*\*\*  $p < 0.001$

These regressions look at US data only. T-bill is the interest rate on 3-month T-bills, at constant maturity. AAA Corporate is the yield on AAA-rated corporate bonds. Inflation is annual. Standard errors are in parentheses. Consistent with our other results, the coefficient of the squared terms on relevant inflation measure is insignificant when the T-bill rate is the dependent variable.

government debt should conform with the Fisher Relationship – at least if a government’s debt is truly default risk free. This presents us with a natural robustness test, since we can estimate the same sort of regressions, same countries and years, with government Treasury Bill rates (from International Financial Statistics).<sup>10</sup> In column 4 of Table 2.3, we show the quadratic estimate for Treasury Bills. The level term is positive and highly significant but the squared term is insignificant. In column 5 we estimate the linear function and that function is plotted in Figure 2.1(d). The plotted function is very close to the 45 degree line, and never near two standard deviations away. Thus, with interest rates on government securities, the data conform almost perfectly to Fisher.

We also rerun all of the regressions adding the data from the US. This increases the number of observations for the Compustat regressions by over 60%. The results are very similar to the original regressions that exclude the US. We also get the same general pattern when running regressions exclusively on the US data, as shown in Table 2.3.

It is possible that the significantly non-linear relationship we have reported is due

<sup>10</sup><http://www.imf.org/external/data.htm>

Table 2.4: Other robustness tests

	(1) Lending rate	(2) Lending rate
Inflation	1.0212 (0.0711)***	
Inflation <sup>2</sup>	-0.0065 (0.0026)**	
Spline1		1.18 (0.1404)***
Spline2		-0.648 (0.2907)**
Constant	6.9317 (0.2674)***	7.203 (0.3912)***
Obs.	1176	1204
Adj. $R^2$	0.52	0.36

\*  $p < 0.05$  , \*\*  $p < 0.01$  , \*\*\*  $p < 0.001$

These regressions use the lending rate data. Column 1 shows quadratic regressions estimated after removal of 28 highly influential data points. Column 2 uses the complete data set and estimates a cubic spline function. Standard errors are in parentheses.

to one or a few outliers. We conducted a test to identify unusual and “influential” data points, using predicted standardized residuals to identify outliers. Observations with standardized residuals less than -2.5 or greater than 2.5 are considered “influential”.<sup>11</sup> We find 23 observations with standardized residuals above 2.5 and 5 observations with standardized residuals below -2.5. Next, we delete these observations and re-estimate the quadratic function. The new results are shown in Table 2.4, column 1. As can be seen by comparing these results with those obtained earlier with the uncensored data set (Table 2.2, column 1), the conclusions are hardly affected, except for a very substantial increase in  $R^2$ . The linear slope coefficient is close to 1.0 in both cases, and in both cases the quadratic term is negative and highly significant.

We also estimate a non-parametric function using the restricted cubic spline method on the full data set. This approach creates variables containing a restricted cubic spline of the variable Inflation (also known as a natural spline). We use three knots.

<sup>11</sup><http://www.ats.ucla.edu/stat/stata/webbooks/reg/chapter2/statareg2.htm>

We do not specify the exact locations of the knot points, but use values based on Harrell (2001)’s recommended percentiles with the additional restriction that the smallest knot may not be less than the fifth-smallest value of Inflation and the largest knot may not be greater than the fifth-largest value of Inflation. The result is in Table 2.4, column 2. The fitted spline function suggests that there is indeed a non-linear relationship that is highly significant. We do not bother to plot it here, because it has the same shape and location as all the others.

The results also hold when we look at interest rate spreads (interest rates less the government rate), but these results might be affected by differences in maturity between government and private rates.

Regarding robustness more generally, we note that all our findings are supported by two completely different international data sets. One is at the country level, the other is at the firm level, and they come from different data providers. We have a powerful natural robustness test of running the same regressions with Treasury Bill data and finding a very different relationship from that with defaultable debt – exactly as predicted by our theory.

## 2.4 Conclusion

Our theoretical results are more general than they might appear. Although we have employed a costly state verification environment, that is not necessary for our theorems to go through. All that is necessary is an environment in which, in bad states of the world, entrepreneurs get nothing and the investor gets the *real assets* of the firm. Other contracting environments such as Hart and Moore (1998) or Holmstrom and Tirole (1997) should give similar results.

Our findings about the Fisher effect also have policy implications. If one takes nominal corporate interest rates from inflationary environments and adjusts for inflation, the real rate estimated in this way is very often negative. Boyd, Levine and

Smith (2001) did this and concluded that inflation was interfering with financial markets; e.g. with negative real rates corporations arguably could not raise funds. Based on the results presented here, however, such an interpretation would be incorrect. For policy purposes, just estimating real rates of interest in the conventional way is not an adequate guide to the effects of inflation. For example, at some point during our sample period, 36 countries experienced negative corporate real interest rates, e.g. negative values for the object “Lending Rate – Annual Inflation”. These negative real rates are observed a total of 87 times out of a total of 1204 country-year observations, or about 7.23%.<sup>12</sup> A development finance researcher might observe such data points and conclude the countries were experiencing market failure. This research suggests that is probably not the correct interpretation. Employing our non-linear regression with the model “ $Lending\ Rate = a + b_1 Inflation + b_2 Inflation^2 + \epsilon$ ” (Table 2.1) for each country/date, we predict the value of the lending rate. Then we calculate the predicted lending rate less annual inflation. These predicted real rates are negative in only 12 instances or approximately 1% of the time. Such issues could be a significant new area of research for finance development scholars.

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<sup>12</sup>The countries include Argentina, Bulgaria, China, Egypt, Estonia, Finland, Gabon, Hong Kong, Iceland, India, Indonesia, Ireland, Jamaica, Jordan, Kenya, Kuwait, Latvia, Liberia, Lithuania, Nigeria, Norway, Oman, Pakistan, Papua New Guinea, Philippines, Qatar, Russian Federation, Slovak Republic, Sri Lanka, Thailand, Trinidad and Tobago, Ukraine, United Kingdom, Venezuela, Vietnam, and Zambia.

## 2.5 Proofs

### 2.5.1 Proof of the contract structure

We know that the optimal contract is debt-like in the real model with risk-neutral agents. We want to show that the same contract structure occurs in the nominal case for each inflation state.

The entrepreneur observes real output  $r \in Y = [L, U]$  and sends a real signal  $y$  to the investor. Then both agents observe the inflation  $p$ . The investor does not observe  $r$  and must choose when to monitor the entrepreneur's output signal. In this case, we impose that the entrepreneur sends the output signal before the agents see inflation, while the decision to monitor can depend on inflation. The notation for monitoring is

$$a(y, p) = \begin{cases} 1 & \text{if audit} \\ 0 & \text{if no audit} \end{cases}$$

Denote the set of signals over which the investor chooses to monitor as  $A(p) = \{y : a(y, p) = 1\}$ . Finally, there is a transfer from the entrepreneur to the investor. In cases with no monitoring, the transfer is  $s(y)$  and is nominal. The investor adjusts this for inflation to arrive at real value  $\frac{s(y)}{p}$ , but we impose that under no monitoring the transfer itself cannot depend on  $p$ . With monitoring, it is  $s_a(r, y, p)$ , with the first argument denoting the true real output and the second argument the entrepreneur's signal. When there is monitoring, the transfer is also in real terms and must depend on  $p$  through the investor's decision of which set of signals to monitor.

To determine what signal to send, the entrepreneur solves

$$\max_y \left\{ E_p \left[ (1 - a(y, p)) \left( r - \frac{s(y)}{p} \right) + a(y, p)(r - s_a(r, y, p)) \right] \right\}$$

To impose truth-telling, the constraints are

$$\begin{aligned}
\forall p, r \in A(p) : \quad r - s_a(r, r, p) &\geq r - s_a(r, y, p) \quad \forall y \in A(p) \\
r - s_a(r, r, p) &\geq r - \frac{s(y)}{p} \quad \forall y \notin A(p) \\
\forall p, r \notin A(p) : \quad r - \frac{s(r)}{p} &\geq r - s_a(r, y, p) \quad \forall y \in A(p) \\
r - \frac{s(r)}{p} &\geq r - \frac{s(y)}{p} \quad \forall y \notin A(p)
\end{aligned}$$

That is, the contract must be structured so that revealing the true output is better for the entrepreneur, regardless of whether output and false signals would induce monitoring or not. We assume the entrepreneur tells the truth whenever indifferent between telling the truth and lying.

We can eliminate the first and third conditions by setting  $s_a(r, y, p) = r$ , as large as possible (punishing the entrepreneur severely for lying when the investor monitors), leaving

$$\begin{aligned}
\forall p, r \in A(p) : \quad r - s_a(r, r, p) &\geq r - \frac{s(y)}{p} \quad \forall y \notin A(p) \\
\forall p, r \notin A(p) : \quad r - \frac{s(r)}{p} &\geq r - \frac{s(y)}{p} \quad \forall y \notin A(p)
\end{aligned}$$

Rewriting,

$$\begin{aligned}
\forall p, r \in A(p) : \quad s_a(r, r, p) &\leq \frac{s(y)}{p} \quad \forall y \notin A(p) \\
\forall p, r \notin A(p) : \quad s(r) &\leq s(y) \quad \forall y \notin A(p)
\end{aligned}$$

The second condition tells us that  $s(y) = \Lambda$ , a constant, for  $y \in Y \setminus A(p) \forall p$ . As we will see, the monitoring sets satisfy  $p_1 > p_2 \implies A(p_1) \subset A(p_2)$ , so  $s$  must be constant outside of the smallest monitoring set.

Now we can consider the full contracting problem. The problem chooses a contract structure to maximize the entrepreneur's return subject to the above constraints and

the constraint that the investor earns the required return  $\alpha$ :

$$\max_{A(p), \Lambda, s_a(\cdot, \cdot, \cdot)} \left\{ E_p \left[ Pr(A(p)) E_r [r - s_a(r, r, p) | r \in A(p)] \right. \right. \\ \left. \left. + (1 - Pr(A(p))) E_r \left[ r - \frac{\Lambda}{p} | r \notin A(p) \right] \right] \right\}$$

subject to

$$s_a(r, r, p) \leq \frac{\Lambda}{p} \quad \forall p, r \in A(p) \\ \alpha \leq E_p \left[ Pr(A(p)) E_r [s_a(r, r, p) - M | y \in A(p)] + (1 - Pr(A(p))) \frac{\Lambda}{p} \right]$$

When the entrepreneur is risk neutral and has limited liability, the investor's constraint binds and the problem reduces to the entrepreneur minimizing monitoring costs

$$\min_{A(p), \Lambda, s_a(\cdot)} \{ E_p [Pr(A(p)) M] \}$$

subject to

$$s_a(r, r, p) \leq \frac{\Lambda}{p} \quad \forall p, r \in A(p) \\ \alpha \leq E_p \left[ Pr(A(p)) E_r [s_a(r, r, p) - M | y \in A(p)] + (1 - Pr(A(p))) \frac{\Lambda}{p} \right]$$

Then  $s_a(r, r, p) = \min \left\{ r, \frac{\Lambda}{p} \right\}$ , otherwise we could increase  $s_a$ , making the investor better off and reducing the size of the monitoring set. As a result,  $s_a(r, r, p) = r$  and  $A(p) = \left\{ r : r < \frac{\Lambda}{p} \right\}$  and if monitoring occurs the investor takes everything. Note that above result regarding the monitoring sets follows from the definition above. Since  $\Lambda$  is constant,  $\frac{\Lambda}{p}$  is decreasing in  $p$ . This framework gives us the result that there is a constant nominal payment to the investor, but the investor adjusts it for inflation and also varies the monitoring set depending on inflation.

## 2.5.2 Proof of theorem 1

Expected monitoring costs in the nominal problem are

$$\mu_N = \frac{1}{2} \int_L^\Lambda M f(r) dr + \frac{1}{2} \int_L^{\frac{\Lambda}{P}} M f(r) dr \quad (2.5.1)$$

$$= \frac{M}{2} \left( \frac{\Lambda - L}{U - L} \right) + \frac{M}{2} \left( \frac{\frac{\Lambda}{P} - L}{U - L} \right) = \frac{M}{2(U - L)} \left( \Lambda - L + \frac{\Lambda}{P} - L \right) \quad (2.5.2)$$

$$= M \left( \frac{\Lambda \left( \frac{P+1}{2P} \right) - L}{U - L} \right) \quad (2.5.3)$$

$$\frac{\partial \mu_N}{\partial P} = \frac{M}{2(U - L)} \left( \frac{\partial \Lambda}{\partial P} \left( 1 + \frac{1}{P} \right) - \frac{\Lambda}{P^2} \right) \quad (2.5.4)$$

$$= \frac{M}{2(U - L)} \left[ \Lambda \left( \frac{\Lambda - (U - M)P}{\Lambda(P^3 + P) - (U - M)(P^3 + P^2)} \right) \left( \frac{P+1}{P} \right) - \frac{\Lambda}{P^2} \right] \quad (2.5.5)$$

$$= \frac{M}{2(U - L)} \left( \frac{\Lambda}{P^2} \right) \left[ \left( \frac{\Lambda - (U - M)P}{\Lambda(P^3 + P) - (U - M)(P^3 + P^2)} \right) (P^2 + P) - 1 \right] \quad (2.5.6)$$

$$= \frac{M}{2(U - L)} \left( \frac{\Lambda}{P^2} \right) \left[ \left( \frac{\Lambda - (U - M)P}{\Lambda(P^2 + 1) - (U - M)(P^2 + P)} \right) (P + 1) - 1 \right] \quad (2.5.7)$$

$$= \frac{M}{2(U - L)} \left( \frac{\Lambda}{P^2} \right) \left[ \frac{\Lambda(P + 1) - (U - M)(P^2 + P)}{\Lambda(P^2 + 1) - (U - M)(P^2 + P)} - 1 \right] \quad (2.5.8)$$

$$= \frac{M}{2(U - L)} \left( \frac{\Lambda}{P^2} \right) \left[ \frac{\Lambda(P + 1) - (U - M)(P^2 + P) - \Lambda(P^2 + 1) + (U - M)(P^2 + P)}{\Lambda(P^2 + 1) - (U - M)(P^2 + P)} \right] \quad (2.5.9)$$

$$= \frac{M}{2(U - L)} \left( \frac{\Lambda}{P^2} \right) \left[ \frac{\Lambda(P + 1) - \Lambda(P^2 + 1)}{\Lambda(P^2 + 1) - (U - M)(P^2 + P)} \right] \quad (2.5.10)$$

$$= \frac{M}{2(U - L)} \left( \frac{\Lambda}{P^2} \right) \left[ \frac{\Lambda(P - P^2)}{\Lambda(P^2 + 1) - (U - M)(P^2 + P)} \right] \quad (2.5.11)$$

$$= \underbrace{\frac{M}{2(U - L)}}_A \underbrace{\left( \frac{\Lambda}{P} \right)}_B \underbrace{\left( \frac{1 - P}{\Lambda(P^2 + 1) - (U - M)(P^2 + P)} \right)}_C \quad (2.5.12)$$

Where we substitute in the definition of  $\frac{\partial \Lambda}{\partial P}$  in equation (2.5.5). The numerator of  $C$  is negative since  $P > 1$ . The denominator is also negative since  $\Lambda < U - M \implies \Lambda(P^2 + 1) < (U - M)(P^2 + 1) < (U - M)(P^2 + P)$ . Therefore,  $A > 0$ ,  $B > 0$ ,  $C > 0$ , so  $\frac{\partial \mu_N}{\partial P} > 0$  and expected monitoring costs are increasing in inflation.



### 2.5.3 Proof of theorem 2

Given inflation  $P \geq 1$  and parameters  $U > L \geq 0$ ,  $U > M > 0$ ,  $\alpha \geq 1$ , the two repayment levels are

$$\begin{aligned}\Lambda &= (U - M) \frac{P^2 + P}{P^2 + 1} - \left( \frac{2P}{P^2 + 1} \right) \sqrt{(U - M)^2 \left( \frac{P + 1}{2} \right)^2 - (P^2 + 1) D} \\ \Lambda_F &= \left( \frac{P + 1}{2} \right) \left[ (U - M) - \sqrt{(U - M)^2 - 2D} \right]\end{aligned}$$

where  $D = \alpha(U - L) - ML + \frac{L^2}{2}$  and  $D > 0$  by assumption. We want to show that  $\Lambda_F(P) > \Lambda(P)$  for all  $P > 1$ .

First note that for the solutions to exist, the discriminant in both cases must be non-negative, and furthermore if the discriminant of the first equation is non-negative, then so is the second. Therefore, we must have

$$\forall P > 1, \quad (U - M)^2 \left( \frac{P + 1}{2} \right)^2 \geq (P^2 + 1) D \implies (U - M)^2 \geq 4 \frac{(P^2 + 1)}{(P + 1)^2} D$$

The function  $\frac{(P^2 + 1)}{(P + 1)^2}$  is increasing in  $P$  for  $P > 1$  with  $\lim_{P \rightarrow \infty} \frac{(P^2 + 1)}{(P + 1)^2} = 1$ . Therefore the parameters must satisfy

$$(U - M)^2 \geq 4D$$

This is a relatively loose restriction; it can be interpreted as simply saying that maximum upper return on the project must be high enough, given the investor's required return and monitoring costs.

Directly comparing the two repayment levels, we have

$$(U - M) \frac{P^2 + P}{P^2 + 1} - \left( \frac{2P}{P^2 + 1} \right) \sqrt{(U - M)^2 \left( \frac{P + 1}{2} \right)^2 - (P^2 + 1) D} < \left( \frac{P + 1}{2} \right) \left[ (U - M) - \sqrt{(U - M)^2 - 2D} \right]$$

$$\begin{aligned}\left( \frac{P + 1}{2} \right) \sqrt{(U - M)^2 - 2D} &< (U - M) \left[ \frac{P + 1}{2} - \frac{P^2 + P}{P^2 + 1} \right] + \left( \frac{2P}{P^2 + 1} \right) \sqrt{(U - M)^2 \left( \frac{P + 1}{2} \right)^2 - (P^2 + 1) D} \\ \left( \frac{P + 1}{2} \right) \sqrt{(U - M)^2 - 2D} &< (U - M) \left[ \frac{(P - 1)^2 (P + 1)}{2(P^2 + 1)} \right] + \left( \frac{2P}{P^2 + 1} \right) \sqrt{(U - M)^2 \left( \frac{P + 1}{2} \right)^2 - (P^2 + 1) D}\end{aligned}$$

Both sides are positive for  $P > 1$  since  $\frac{(P - 1)^2 (P + 1)}{2(P^2 + 1)} > 0$ , so squaring both sides preserves the inequality (note also that the expression below holds with equality at  $P = 1$ , when the two repayment

levels are the same):

$$\begin{aligned} \left(\frac{P+1}{2}\right)^2 \left((U-M)^2 - 2D\right) &< (U-M)^2 \left[\frac{(P-1)^2(P+1)}{2(P^2+1)}\right]^2 + \left(\frac{2P}{P^2+1}\right)^2 \left((U-M)^2 \left(\frac{P+1}{2}\right)^2 - (P^2+1)D\right) \\ &+ 2(U-M) \frac{P(P-1)^2(P+1)}{(P^2+1)^2} \sqrt{(U-M)^2 \left(\frac{P+1}{2}\right)^2 - (P^2+1)D} \end{aligned}$$

Rearranging,

$$\begin{aligned} (U-M)^2 \left[\left(\frac{P+1}{2}\right)^2 - \left(\frac{(P-1)^2(P+1)}{2(P^2+1)}\right)^2 - \left(\frac{2P}{P^2+1}\right)^2 \left(\frac{P+1}{2}\right)^2\right] \\ < D \left[2\left(\frac{P+1}{2}\right)^2 - \left(\frac{2P}{P^2+1}\right)^2 (P^2+1)\right] + 2(U-M) \left[\frac{P(P-1)^2(P+1)}{(P^2+1)^2}\right] \sqrt{(U-M)^2 \left(\frac{P+1}{2}\right)^2 - (P^2+1)D} \end{aligned}$$

$$\begin{aligned} (U-M)^2 \left[\frac{P(P-1)^2(P+1)^2}{(P^2+1)^2}\right] &< D \left[\frac{(P-1)^2(P^2+4P+1)}{2(P^2+1)}\right] + \\ &2(U-M) \left[\frac{P(P-1)^2(P+1)}{(P^2+1)^2}\right] \sqrt{(U-M)^2 \left(\frac{P+1}{2}\right)^2 - (P^2+1)D} \\ (U-M)^2 &< D \left[\frac{(P^2+1)(P^2+4P+1)}{2P(P+1)^2}\right] + (U-M) \left[\frac{2}{P+1}\right] \sqrt{(U-M)^2 \left(\frac{P+1}{2}\right)^2 - (P^2+1)D} \\ (U-M)^2 &< D \left[\frac{(P^2+1)(P^2+4P+1)}{2P(P+1)^2}\right] + (U-M) \sqrt{(U-M)^2 - 4\frac{(P^2+1)}{(P+1)^2}D} \\ (U-M)^2 &< DA(P) + (U-M) \sqrt{(U-M)^2 - 4B(P)D} \end{aligned}$$

where we define  $A(P) = \frac{(P^2+1)(P^2+4P+1)}{2P(P+1)^2}$  and  $B(P) = \frac{(P^2+1)}{(P+1)^2}$

For  $P > 1$ ,  $A(P) > \frac{3}{2}$  and is increasing with  $\lim_{P \rightarrow \infty} A(P) = \infty$ , and  $B(P) > \frac{1}{2}$  and is increasing with  $\lim_{P \rightarrow \infty} B(P) = 1$ . To find an appropriate parameter restriction such that the inequality holds, it would be nice to minimize the value of the RHS by minimizing  $A(P)$  and maximizing  $B(P)$ . This would imply that the inequality holds for all value of  $P$ . However, that causes the inequality never to be true. Instead, break the domain of  $P$  into two parts,  $P \in (1, 3)$  and  $P \in [3, \infty)$ . The range of  $A(P)$  in the first region is  $(\frac{3}{2}, \frac{55}{24})$  and in the second is  $[\frac{55}{24}, \infty)$ . For  $B(P)$  the corresponding ranges are  $(\frac{1}{2}, \frac{5}{8})$  and  $[\frac{5}{8}, \infty)$ . We show that the inequality holds in each region for a particular parameter restriction, and then take the strongest parameter restriction to prove the overall result.

Minimizing the RHS of the last inequality in region 1 by taking the smallest value of  $A(P)$  and the largest value of  $B(P)$ ,

$$\begin{aligned}
(U-M)^2 &< \frac{3}{2}D + (U-M)\sqrt{(U-M)^2 - \frac{5}{2}D} \\
(U-M)^2 - \frac{3}{2}D &< (U-M)\sqrt{(U-M)^2 - \frac{5}{2}D} \\
(U-M)^4 + \frac{9}{4}D^2 - 3D(U-M)^2 &< (U-M)^4 - \frac{5}{2}D(U-M)^2 \\
\frac{9}{4}D^2 &< \frac{1}{2}D(U-M)^2 \\
(U-M)^2 &> \frac{9}{2}D > 4D
\end{aligned}$$

The assumption that  $(U-M)^2 \geq 4D$  means that the inequality is satisfied. In region 2, using the smallest value of  $A(P)$  and the largest value of  $B(P)$ ,

$$\begin{aligned}
(U-M)^2 &< \frac{55}{24}D + (U-M)\sqrt{(U-M)^2 - 4D} \\
(U-M)^2 - \frac{55}{24}D &< (U-M)\sqrt{(U-M)^2 - 4D} \\
(U-M)^4 + \frac{3025}{576}D^2 - \frac{55}{12}D(U-M)^2 &< (U-M)^4 - 4D(U-M)^2 \\
\frac{3025}{576}D^2 &< \frac{7}{12}D(U-M)^2 \\
(U-M)^2 &> \frac{3025}{336}D
\end{aligned}$$

This is a tighter parameter restriction than previously. Note that squaring in step two requires  $(U-M)^2 > \frac{55}{24}D > 4D$ . If this stronger (but still reasonable) assumption holds, then  $\Lambda_F(P) > \Lambda(P)$  for all  $P > 1$ , provided  $(U-M)^2 \geq \frac{3025}{336}D$ .

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